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Application of Analytic Hierarchy Process to Prioritize Urban Transport Options – Comparative Analysis of Group Aggregation Methods

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The present study presents a comparative analysis of different group aggregation methods adopted in AHP by testing them against social choice axioms with a case study of Delhi transport system. The group aggregation (GA) methods and their correctness were tested while prioritizing the alternative options to achieve energy efficient and less polluting transport system in Delhi

It was observed that among all group aggregation methods, geometric mean method (GMM) - the most widely adopted GA method of AHP - showed poor performance and failed to satisfy the most popular "pareto optimality and non-dictatorship axiom" raising questions on its validity as GA method adopted in AHP. All other group aggregation methods viz. weighted arithmetic mean method with varying weights and equal weights (WAMM, WeAMM) and arithmetic mean of individual priorities (AMM) resulted in concurring results with the individual member priorities.

This study demonstrates that W_eAMM resulted in better aggregation of individual priorities compared to WAMM. Comparative analysis between individual and group priorities demonstrates that the arithmetic mean (AMM) of priorities by individual members of the group showed minimum deviation from the group consensus making it the most suitable and simple method to aggregate individual preferences to arrive at a group consensus.

Key words: AHP, decision making, GMM, group aggregation, transportation, WAMM

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1. Introduction

Priority theory is a well established subject with wide range of applications to different sectors. Most of the priority theory based methodologies follow either quantitative or qualitative criteria to attribute priorities. Thomas L. Saaty's Analytic Hierarchy Process (AHP) developed in late 80's, prioritizes alternatives based on qualitative and quantitative criteria. AHP combines deductive approach and systems approach of solving problems into one integrated logical framework and this makes it that much more effective in priority setting.

AHP is known for its potential in group aggregation. In spite of being used predominantly, geometric mean and arithmetic mean methods are under consistent debate for their validity in group aggregation (Aczel and saaty, 1983; Basak and Saaty, 1993; Richelson, 1981). In particular, geometric mean method (GMM) was found causing rank reversal in group aggregation (Kirkwood, 1979) and failing to satisfy few obvious social choice axioms. It was evident from the literature on group aggregation and decision making that any group aggregation methodology needs to be checked against certain social choice axioms. In spite of the fact that GA methods posing problems, there exist no comprehensive comparative analysis of GA methods adopted in AHP to identify which one proves better. Such comparative analysis and empirical evidences are grossly missing in the literature.

In the present study, the group aggregation methods commonly employed in AHP are tested against the standard social choice axioms and a comparative analysis has been carried out. Delhi urban transport system was selected as a case in which AHP has been applied to prioritize the selected alternative options for energy efficient and less polluting transport system in Delhi. Prioritization has been carried out by using four different group aggregation methods viz. geometric mean method (GMM), weighted arithmetic mean method with equal weights (W_eAMM), weighted arithmetic mean method with varying weights (WAMM) and arithmetic mean of individual priorities (AMM) to make a comparison among them and check them against social choice axioms. Subjective comparisons provided by a group of individuals encompassing different key departments and actors of transport sector adds to the strength of this exercise of prioritizing the transportation options and comparison of GA methods adopted in AHP.

2. Objective

Objective of the present study is to make comparative analysis of GA methodologies adopted in AHP and assess their potential for effective group aggregation by checking them against social choice axioms with a case study of prioritizing alternative transportation options for Delhi transport system.

3. Group aggregation and AHP

This section presents a brief outline of developments on group aggregation and analytic hierarchy process. Most of the early works on aggregation of individual priorities are based on utility theory. Aggregation of individual preferences to obtain a group consensus has started as early as in 1951 with the "Impossibility Theorem" of Arrow. Keeney in 1976 had specified a set of sufficient conditions for a cordinal social welfare function to have the weighted additive form. In further development, Mirkin (1979) has developed an eigen vector based method to determine group evaluation using constant coefficients which measure the change in evaluation of a member due to interactions with other members of the group. Korhonen and Wallenius (1990) have demonstrated a computer aided interactive mathematical programming technique for solving group decision problems.

In the year 1980, Saaty had developed analytic hierarchy process (AHP) for group decision making. AHP, unlike other decision-making processes, has the capability of handling both qualitative and quantitative parameters. The three principles of guidance in AHP are decomposition, comparative judgement and synthesis of priorities (Saaty, 1980, Saaty, 1990). AHP model is an effective tool for priority setting because AHP combines deductive approach and systems approach of solving problems into one, integrated logical framework. It integrates qualitative and quantitative criteria and arrives at priorities of alternatives. The fundamental principle of AHP is the "pair-wise comparison of different variables which are given numerical values for their subjective judgements on relative importance of each of the variable following a hierarchy and coming out with assigning relative weights to those variables". This process breaks down a complex and unstructured situation into components forming a hierarchy. This technique has been used by many researchers for wide range of applications (Hannan, 1983). Saaty had presented a thorough discussion (Saaty, 1986; Saaty, 1994) on several theoretical and practical aspects of group decision-making using AHP.

Many methodologies viz. consensus voting, combined individual judgements (Harker and Vargas, 1987), geometric mean method (Aczel and Saaty, 1983), weighted arithmetic mean method are tried for group aggregation. Most common group aggregation methods adopted in AHP are geometric mean method (GMM) and weighted arithmetic mean method (WAMM). All the above GA methods have their limitations in group aggregation. Exponential function in GMM magnifies even the slightest deviation in individual preferences resulting in poor sensitivity. According to Zahir (1999), larger groups are more likely to get affected by this. In weighted arithmetic mean methods deriving weights 'w' poses a potential problem. There is another method of aggregating individual preferences in AHP, which includes the actors as one of the levels of AHP hierarchy (Aczel and Saaty, 1983). In such cases the large scale hierarchy interferes with the rank preservation. In spite of having problems with all the above GA methods, a comprehensive comparative analysis to assess and compare their potential in aggregating individual priorities to get group consensus is grossly missing in the literature.

3.1 Social choice axioms

Any decision derived from a group of individuals has to satisfy a set of social choice axioms. Early works of Arrow (1951), "the impossibility theorem", has been a major influence in this area. Works of Richelson (1981), Plott (1976), Benjamin et al., (1992) etc., are few examples of further efforts in line with Arrow's work. Richelson has evaluated many social choice functions such as 'Simple Plurality' and the 'Borda Counts' using 20 different social choice axioms. Plott (1976) tried to present the overview of axiomatic social choice theory. The importance of social choice axioms in group aggregation is well accepted and among the 20 social choice axioms discussed by Richelson, *universal domain axiom, pareto optimality axiom, independence of irrelevant alternative axiom, non-dictatorship axiom and recognition axioms* are the most popular and commonly used axioms (Keeney 1976; Mirkin 1979).

Among the axioms listed above, pareto and non-dictatorship and recognition axioms are widely accepted axioms and any group aggregation process is expected to satisfy them. Although the axiom "Universal domain" seems reasonable, it has been claimed that extreme divergence of opinions among group members should be avoided. Independence of irrelevant alternative axiom has been under discussion and criticism by many researchers (Hanssan, 1969). Hence, pareto and non-dictatorship axioms are considered for the comparative analysis of GA methods in the present study.

4. Methodology

4.1 Urban transport system in Delhi

Delhi, the capital city of India has been facing tremendous growth in travel demand and vehicular population resulting out of increased urbanization, population, economic growth and improved road network. Delhi roads are dominated by personalized modes of transport viz. 2-wheelers and cars (IGIDR, 2000). This may be due to the absence of an efficient public transport system. Uncontrolled vehicular growth resulted in increase in air pollution making the Indian capital city, the fourth most polluted city in the world. This is an alarming situation requiring immediate action to minimize the energy demands from urban transport sector and also to control the pollution. No single option would result in improving the situation considerably. And also various actors involved may show different priorities over the available alternative options. Hence, it is essential to apply multi-criteria decision making processes to arrive at group priorities for the question of which alternative option should be given more weight in implementation to achieve improved transport system, which is energy efficient and less polluting.

4.2 Development of framework for AHP

As the roads of Delhi are more dominated by 2-Wheelers and cars, the following options have been selected to achieve sustainable transportation.

Option - I:	Replacing 2-stroke 2-wheelers by 4-stroke 2-wheelers	(AI)
Option - II:	Converting conventional fuel cars by CNG cars	(AII)
Option - III:	Converting conventional fuel buses by CNG buses	(AIII)

As different actors involved may have different priorities for options, ranking needs to be done by a group of actors. This should include all those categories of people who have influence over it either directly or indirectly as shown below:

a. Environmental experts
b. Energy experts
c. Users
d. Federal department/Policy maker
e. Automobile association
f. Automobile research institute
g. Local level implementing agency

To achieve better ranking, it is important to select the list of criteria based on which the comparative judgements are made. The following criteria have been selected based on the options that are selected and also the goal of the hierarchy "selection of alternative options for sustainable urban transport in Delhi".

1. Energy efficiency (Energy) (C1)
2. Emission reduction potential (Environment) (C2)
3. Economic feasibility (Cost) (C3)
4. Technological preparedness (Technology) (C4)
5. Implementability/Adaptability (C5)
6. Barriers to the implementation of these options (Barriers) (C5)

4.2.1 Construction of AHP tree

This section describes the construction of the hierarchical tree for current problem under consideration.

Goal: Goal of the process is to prioritize a set of alternatives for the improvement of transport system in Delhi.

Criteria: Criteria constitute the first level of the hierarchy and the elements at this level include Energy, Environment, Cost, Technology, Adaptability and Barriers.

Alternatives: Alternatives viz. replacing 2-stroke 2-wheelers by 4-stroke 2-wheelers, conversion of conventional fuel cars to CNG cars, conversion of conventional fuel buses to CNG buses represent the second level in the current hierarchy. Figure 1 gives the graphical view of the hierarchy tree.

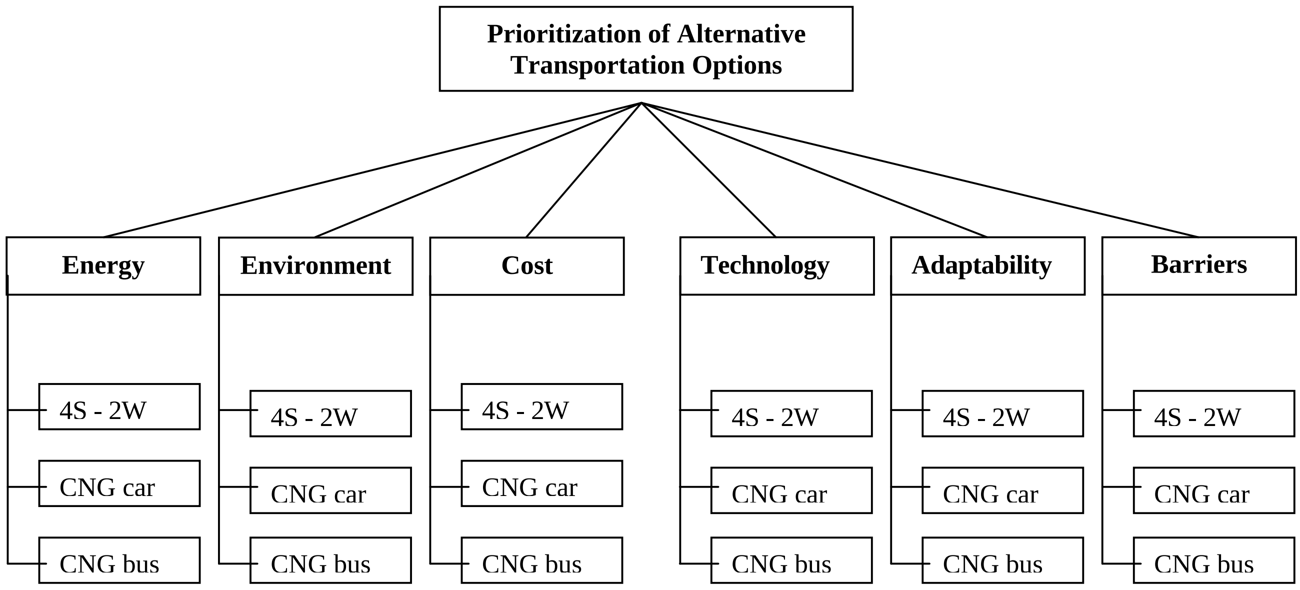

Fig. 1. AHP hierarchy tree for the prioritization of alternative transportation options

This tree is made of three quantitative criteria and three qualitative criteria. Among the list of criteria Cost, Energy and Environment fall under the category of quantitative parameter and the other three namely Technology, Adaptability and Barriers are qualitative. Each one needs essentially a separate methodology for their quantification and subsequent prioritization.

4.3 Quantitative criteria

4.3.1 Energy

Prioritization of various options was done by finding out their energy saving potential. Total energy demand of a particular travel mode of any particular option (for instance, total energy demand of cars in the case AII) was considered to calculate the energy saving potential by using the following equation:

$$ESP = 1 - \frac{E_{jt_{alt}}}{E_{jt_{old}}} \qquad \text{(i)}$$

where,

ESP Energy saving potential

$E_{jt\text{-}alt}$ Energy requirement of the travel mode 'j' in alternative technology in the year 't'

$E_{jt\text{-}old}$ Energy requirement of the travel mode 'j' in existing technology in the year 't'

Energy requirement of each option was determined by considering the total PKM catered by the respective mode of transport of the option under consideration and the respective energy intensity factor. Normalization technique is used to arrive at priorities of alternative options under each quantitative criteria namely energy, environment and cost.

4.3.2 Environment

Prioritization of alternative options with reference to the environmental criteria was done by calculating emission reduction potential (ERP) of each alternative.

$$ERP = 1 - \frac{P_{it_{alt}}}{P_{it_{old}}} \qquad \text{(ii)}$$

where,

ERP	Emission reduction potential
$P_{it\text{-}alt}$	Emission of pollutant type 'i' in the alternative technology in the year 't'
$P_{it\text{-}old}$	Emission of pollutant type 'i' in the existing technology in the year 't'

4.3.3 Cost

For each option cost is represented by the life cycle operating cost (LCC). LCC of each alternative option was determined by using the following formula:

$$LCC = \frac{Levelised\ cost}{PKM_j} \qquad \text{(iii)}$$

where,

LCC	Life cycle operating cost
LC	Levelised cost of the option (includes capital cost, operation costs, O&M costs, taxes and subsidies etc.)
PKM_{jt}	PKM covered by travel mode 'j' in the alternative option for the year 't'

4.4 Qualitative criteria

Subjective judgements from the group members are collected in terms of pairwise judgements. A specially designed questionnaire was used to get the pairwise comparison matrices. AHP based decision software named "Expert Choice" is used in certain cases to get priorities.

4.4.1 Questionnaire design

Questionnaire survey was adopted to complete the pairwise matrices. A specially designed questionnaire was given to all the respondents in the group and were given sufficient time to send back their responses. Questionnaire survey has been used to get priority matrices for criteria, actors and alternatives under qualitative criteria.

Priorities of alternative options based on qualitative criteria are calculated in four methods by adopting GMM, W_eAMM, WAMM and AMM. Final priorities of alternative options are determined as four cases by forming the final matrices with quantitative criteria and qualitative criteria by each GA method.

5. Results and Analysis

Analysis has been carried out using four group aggregation methods. In the present case of hierarchy, alternative options provide the lowest level with criteria as an intermediate level and goal at the top level. As in AHP the priorities attributed to the lower level of hierarchy adds to the prioritization of upper levels, prioritization of lower level is carried out first to attribute priorities to the alternative options with respect to each criteria.

5.1 Quantitative criteria

5.1.1 Energy

LEAP model (Long Range Energy Alternative Planning) was used to estimate the energy demand of the vehicles of different modes for the year 1998. Table 1 provides the energy demand of all options under consideration.

Table 1
Energy demands of various alternative technologies calculated by using LEAP model

Travel mode	Total PKM catered by the mode under consideration (million)	Total energy demand of mode 'J' (Million GJ)
2-wheelers –2-stroke	11.32	6.11
2-wheelers –4-stroke	11.32	4.19 (31.42%) ↓
Cars –petrol	18.17	19.60
Cars – diesel	18.17	--
Cars – CNG	18.17	11.23 (42.70%) ↓
Taxi – petrol	0.62	1.606
Taxi – diesel	0.62	--
Taxi-CNG	0.62	1.025 (36.18%) ↓
Bus – diesel	39.02	12.17
Bus- CNG	39.02	11.78 (3.20%) ↓

In the above table, figures in parenthesis indicate the percentage change in energy demand for alternative option with respect to the base case. The downward arrows indicate percentage fall in energy demand. Energy saving potential (ESP) was calculated and the priorities of the three alternative options under consideration with respect to the energy criteria are determined by adopting normalization technique. Table 2 presents the energy saving potential (ESP) and priorities of the three alternatives with respect to the energy criteria.

Table 2
Priorities of all alternatives under the criteria "Energy"

Alternative Option	Energy saving potential (ESP)	Priority
4-S 2-wheelers	0.314	0.4089
CNG Cars	0.422*	0.5494*
CNG Buses	0.032	0.0416

* Car and Taxi have been added together

5.1.2 Environment

Emission of all pollutants under consideration (CO_2, CO, SO_x, NO_x, HC, TSP, Pb) was calculated both for base case and alternative options. Table 3 presents the reduction in total emission levels of each pollutant in the alternative options.

Table 3
Reduction in overall emission levels of Delhi for different alternative options

Option	Total annual emission of pollutants ('000 t)						
	CO_2	CO	SOx	NOx	HC	TSP	Pb
2-wheelers							
2-stroke (base case)	3.48	173.12	6.77	50.02	59.02	10.04	0.077
4-stroke (alternative case)	3.35	173.16	6.67	52.00	31.50	7.45	0.071
Cars							
Gasoline (base case)	3.48	173.12	6.77	50.02	59.02	10.04	0.077
CNG (alternative case)	3.57	101.25	4.39	41.44	52.06	7.92	0.039
Buses							
Diesel (base case)	3.48	173.12	6.77	50.02	59.02	10.04	0.077
CNG (alternative case)	3.99	161.27	4.42	31.69	57.05	8.16	0.077

All three options showed significant influence on different pollutants and their levels in overall pollution levels in Delhi. However, unit improvement of pollution level in the respective mode of the option needs to be calculated to get emission reduction potential (ERP) of each option. For instance, pollution reduced by using CNG cars instead of gasoline and diesel cars per PKM traveled demonstrates the ERP better. Table 4 presents the unit emission reduction of each pollutant in the respective mode of transport under base case as well as alternative options.

ERP of all alternative options for different kind of pollutants was calculated using the formula given in the methodology. ERP approaching unity indicates better potential of the alternative option.

Table 4
Emission reduction of each mode of transport in respective option per unit output

Option	Fuel type	Emission (g)/PKM						
		CO_2	CO	SOx	NOx	HC	TSP	Pb
2-wheelers	2-stroke	37.70	4.53	0.0257	0.0545	2.8251	0.2726	0.002
	4-stroke	25.83	4.53	0.0177	0.2128	0.3939	00437	0.001
Cars	Gasoline	73.19	3.95	0.1306	0.5495	0.3833	0.1164	0.002
	CNG	78.01	0.0042	0	0.0669	0	0	0
Buses	Diesel	22.89	0.3055	6E-05	0.5054	5E-5	4.8E-5	0
	CNG	35.84	0.0019	0	0.0307	0	0	0

Table 5
Emission reduction potential (ERP) of different alternatives in Delhi

Option	Emission reduction potential (base year)						
	CO_2	CO	SO_x	NOx	HC	TSP	Pb
4-stroke 2-wheelers	0.3148	-0.0008	0.315	-2.904	0.861	0.839	0.313
CNG cars	-0.066	0.998	1.000	0.878	1.000	1.000	1.000
CNG buses	-0.565	0.994	1.000	0.939	1.000	1.000	0.000

Different options show potential in controlling different pollutants. Adding up all the pollutants would represent the overall emission reduction potential. However, domination of pollutants is location specific. For instance, TSP, HC and SOx concentrations typically dominate Delhi air pollution. Therefore, potential of alternative options in controlling these pollutants should be given more weight. Hence, the following weights are assigned to each of the pollutants under consideration. This weight assigning process was done by adopting single actor approach.

Pollutant	TSP	CO	Nox	SOx	HC	Pb
Weight	0.300	0.100	0.100	0.200	0.200	0.100

Overall ERP of each alternative option has been calculated and is presented in Table 6. Priorities of each alternative with respect to the environment criterion are presented in the table below.

Table 6
Priorities of different alternatives under the criteria "Environment"

Option	Weighted ERP	Priority
4-stroke 2-wheelers	0.2277	0.1079
CNG cars	0.9876	0.4684
CNG buses	0.8933	0.4236

5.1.3 Cost

Cost effectiveness of each option was assessed in terms of life cycle operation cost (LCC) per unit of pollution reduced. Total pollution load of all local pollutants together was considered to find out the cost effectiveness. Priorities of each alternative under the cost criterion are calculated by normalizing the unit abatement costs. An increase in the cost due to pollution reduction was given a positive sign where as decrease in cost due to adaptation of less energy intensive system resulting reduction in cost was given a negative sign. Table 7 presents the LCC of each alternative, unit abatement cost and priorities of all three alternatives under the cost criterion.

Table 7
Priorities of three alternatives under the criteria "Cost"

Option	LCC (Rs/pkm)*	Abatement cost (Rs/Kg)	Priority
4-stroke 2-wheeler	1.2468	-33.5	0.244
CNG car	1.9218	-104.4	0.743
CNG bus	0.0747	0.45	0.003

* 1 USD ≈ 49 Indian rupees

Following is the matrix form of priorities of all alternatives under quantitative criteria energy, environment and cost:

	Energy	Environment	Cost
4-stroke 2-wheelers	0.409	0.108	0.244
CNG car	0.549	0.468	0.743
CNG bus	0.042	0.424	0.003

5.2 Qualitative criteria

This section presents the prioritization of alternatives based on qualitative criteria viz. availability of technology, adaptability and barriers. Pairwise judgements of different actors for alternatives under different criteria are aggregated to get the pairwise comparison matrix of the group. Weights for alternative so derived are added to the weightage matrix derived from quantitative criteria and final weights were derived. The group aggregation of the individual priorities under quantitative criteria was carried out in four different methods.

5.2.1 GMM

The individual pairwise matrices provided by the group members for the alternatives in each qualitative criteria are used to get the aggregated pairwise matrix. Geometric mean was calculated by using the formula:

$$\left(\prod_{k=1}^{n} a_{ij}^{k}\right)^{1/n} \qquad \text{(iv)}$$

where, n is the number of members and a_{ij} is the preference of a member for element 'i' over 'j'.

Pairwise matrices of the group for all three alternatives under three criteria namely technology, adaptability and barriers calculated by GMM and are presented below. Pairwise matrix of the group for the prioritization of criteria was also calculated using GMM and presented here.

$$\begin{bmatrix} 1 & 0.891 & 0.257 \\ 1.122 & 1 & 0.327 \\ 3.87 & 3.046 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0.41 & 0.59 \\ 2.42 & 1 & 0.76 \\ 1.96 & 1.29 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0.59 & 0.59 \\ 2 & 1 & 0.62 \\ 1.68 & 1.57 & 1 \end{bmatrix}$$

Pairwise matrix of the group w.r.t. 'Technology'	Pairwise matrix of the group w.r.t. 'Adaptability'	Pairwise matrix of the group w.r.t. 'Barriers'

$$\begin{bmatrix} 1 & 1.02 & 0.91 & 0.64 & 0.86 & 0.99 \\ 0.96 & 1 & 1.24 & 0.51 & 0.59 & 0.72 \\ 1.09 & 0.80 & 1 & 0.84 & 0.63 & 0.77 \\ 1.55 & 1.93 & 1.17 & 1 & 0.88 & 1.32 \\ 1.14 & 1.67 & 1.57 & 1.12 & 1 & 1.69 \\ 1 & 1.37 & 1.29 & 0.74 & 0.58 & 1 \end{bmatrix}$$

Pairwise matrix of the group for criteria

Eigen vectors are calculated for all the above matrices and also the respective weightage matrices, which are shown below. w_c is the weightage matrix for the criteria and w_{c4}, w_{c5} and w_{c6} are the weightage matrices of the three qualitative criteria technology, adaptability and barriers, respectively.

$$w_{c4} = \begin{bmatrix} 0.632 \\ 0.170 \\ 0.198 \end{bmatrix}, \quad w_{c5} = \begin{bmatrix} 0.429 \\ 0.186 \\ 0.385 \end{bmatrix}, \quad w_{c6} = \begin{bmatrix} 0.446 \\ 0.227 \\ 0.326 \end{bmatrix}, \quad w_c = \begin{bmatrix} 0.147 \\ 0.132 \\ 0.139 \\ 0.207 \\ 0.219 \\ 0.157 \end{bmatrix}$$

Consistency ratio was found to be in a valid range as per Saaty's analytic hierarchy process (Saaty, 1990).

With the weights of the alternatives under the three qualitative criteria, weightage matrix for the criteria (shown above) and weights of alternatives under three quantitative criteria (shown in 5.1), hierarchy tree takes the form as shown in Figure 1. Following are the weightage matrices of the alternatives (3x6) and criteria (1x6) for the final priority derivation.

$$\begin{bmatrix} 0.244 & 0..409 & 0.107 & 0.632 & 0.429 & 0.446 \\ 0.743 & 0.549 & 0.468 & 0.170 & 0.186 & 0.227 \\ 0.003 & 0.041 & 0.423 & 0.198 & 0.385 & 0.326 \end{bmatrix} \qquad \begin{bmatrix} 0.147 \\ 0.132 \\ 0.139 \\ 0.207 \\ 0.219 \\ 0.157 \end{bmatrix}$$

Matrix of final priorities for all the alternatives was determined by applying matrix algebra. Priorities of three alternatives given by the group are shown below:

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \longrightarrow \begin{bmatrix} 0.375 \\ 0.213 \\ 0.413 \end{bmatrix}$$

5.2.2 Weighted arithmetic mean method

Following equation was adopted to determine the group consensus matrix using WAMM:

$$P_g A_j = \sum_{1}^{n} W_i P_i (A_j) \qquad \text{(v)}$$

where,

P_gA_j	group priority of alternative A_j
$P_i(A_j)$	priority of A_j given by member E_i
W_i	weight to be given to the preference of E_i
n	number of group members

In the case of *W_eAMM* equal weights were assumed for all the qualitative criteria. Hence, the above equation takes the following form.

$$P_g A_j = \sum_{1}^{n} P_i (A_j) \Big/ n \qquad \text{(vi)}$$

Weightage matrix for group members (w_i) given by the group members themselves was determined as in the case of weight derivation for criteria and alternative options. This process gives the w_i matrix which was used in WAMM. Similar process of vector algebra is followed as in the case of GMM to arrive at the final weightage matrices under W_eAMM and WAMM for the three alternatives options.

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \longrightarrow \begin{bmatrix} 0.266 \\ 0.262 \\ 0.471 \end{bmatrix} \qquad \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \longrightarrow \begin{bmatrix} 0.316 \\ 0.231 \\ 0.453 \end{bmatrix}$$

WeAMM WAMM

5.2.3 AMM

Priorities of alternatives given by the individual members of the group were determined using the Expert Choice software. Priorities for the three alternative options given by individual members of the group are presented in Table 8. Final priorities of alternatives given by the individual members are aggregated by arithmetic mean method to arrive at the group consensus. Priorities of alternatives given by the group are as shown below:

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \longrightarrow \begin{bmatrix} 0.372 \\ 0.208 \\ 0.421 \end{bmatrix}$$

Table 8
Priorities for the three different alternatives provided by individual members of the group

Option	Priorities given by individual members of the group					
	M_1	M_2	M_3	M_4	M_5	M_6*
4-s 2-wheelers	0.349 (II)	0.366 (II)	0.423 (II)	0.228 (III)	0.492 (I)	-
CNG cars	0.232 (III)	0.176 (III)	0.148 (III)	0.329 (II)	0.155 (III)	-
CNG buses	0.420 (I)	0.458 (I)	0.429 (I)	0.443 (I)	0.353 (II)	-

* M_6 - inconsistency is beyond the allowable limit of 0.1
** figures in parenthesis indicate the ranking

5.3 Comparative analysis of GA methods

Priorities of alternative options determined using different GA methods was found following different patterns. Table 9 presents the comparative analysis of different group aggregation methodology adopted in AHP.

Table 9
Priorities for the three different alternatives derived from four different group aggregation methods

Option	Priorities			
	GMM	W_eAMM	WAMM	AMM
4-stroke bikes	0.213 (III)	0.266 (II)	0.316 (II)	0.372 (II)
CNG cars	0.375 (II)	0.262 (III)	0.231 (III)	0.208 (III)
CNG buses	0.413 (I)	0.471 (I)	0.453 (I)	0.421 (I)

In the case of GMM, CNG bus received the top priority of the group followed by CNG car. Where as weighted arithmetic mean method showed slight difference in priorities with CNG bus on top followed by 4-stroke 2-wheelres. Attributing weights to various actors did not show much of difference on final ranking of options. Arithmetic mean of individual priorities has followed WAMM.

GMM	→	A3 > A2 > A1	(A2 > A1)
WeAMM	→	A3 > A1 > A2	(A1 > A2)
WAMM	→	A3 > A1 > A2	
AMM	→	A3 > A1 > A2	

GMM showed its inability in preserving rank. It was explained by Saaty (Saaty, 1990; Saaty, 1994) that the deviation of the group consensus from the individual members can be explained by the consistency index. He explains that if the consistency index of individual members of the group in giving pairwise comparisons is less than 0.1, the deviation could be minimized. However, in the present study it was found that in spite of the individuals being within the Saaty's consistency limits, geometric mean method of group aggregation failed to preserve the rank. Group consensus arrived at using all GA methods except GMM is following the individual actor choices. When the individual preferences are aggregated using GMM there was a rank reversal between A1 and A2. Figure 2 shows the preferences given by the individuals and the group consensus in a graphical form.

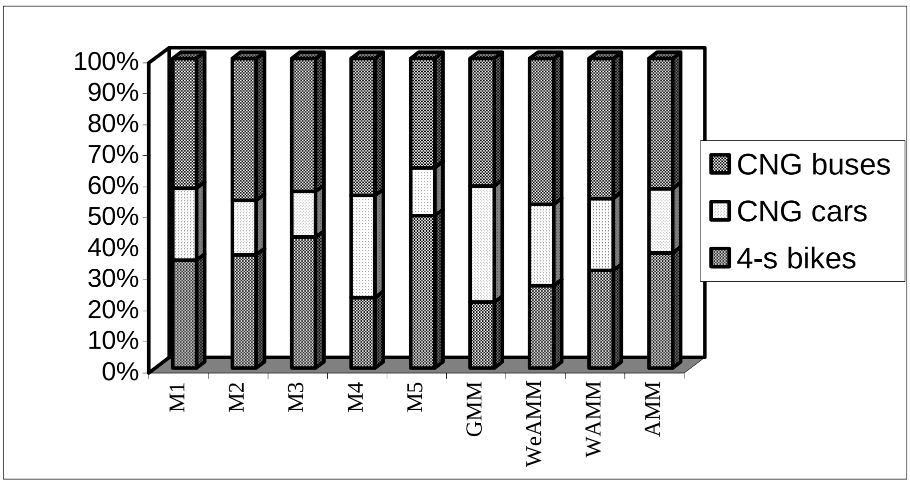

Fig. 2. Priorities of three different alternatives given by individuals as well as derived from four different GA methodology

From the above results it is apparent that GMM failed to satisfy the pareto optimality axiom, which is a well accepted axiom for group aggregation. Figure 3 to 6 demonstrates the deviation of individual member priorities from group consensus arrived using different GA methodology.

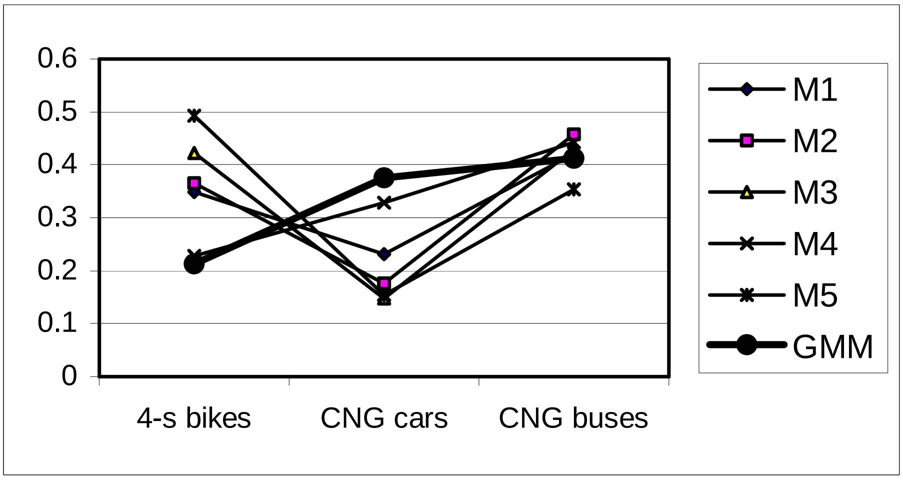

Fig. 3. Deviation of individual member priorities from group consensus (GMM)

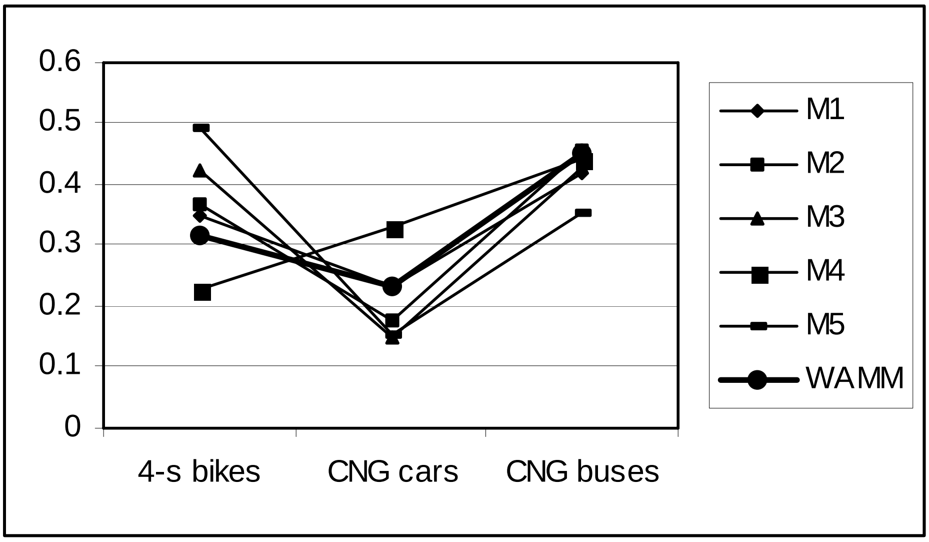

Fig. 4. Deviation of individual member priorities from group consensus (WAMM)

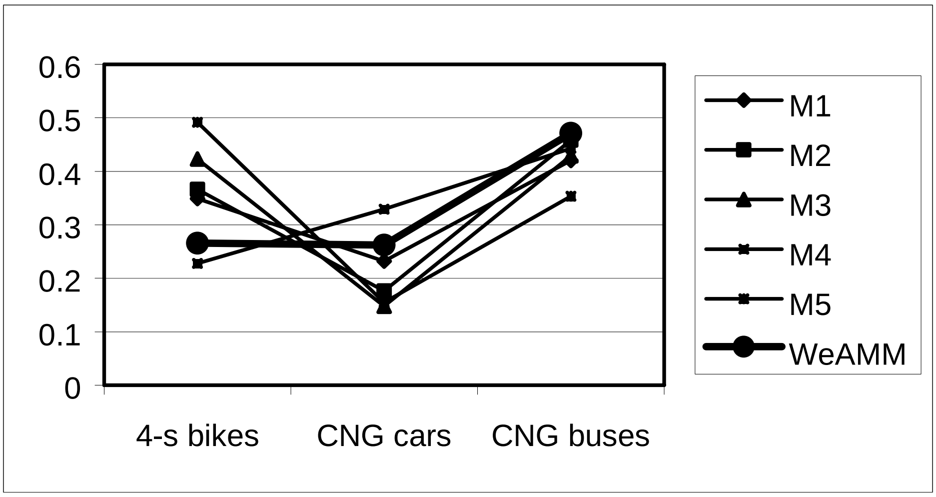

Fig. 5. Deviation of individual member priorities from group consensus (WeAMM)

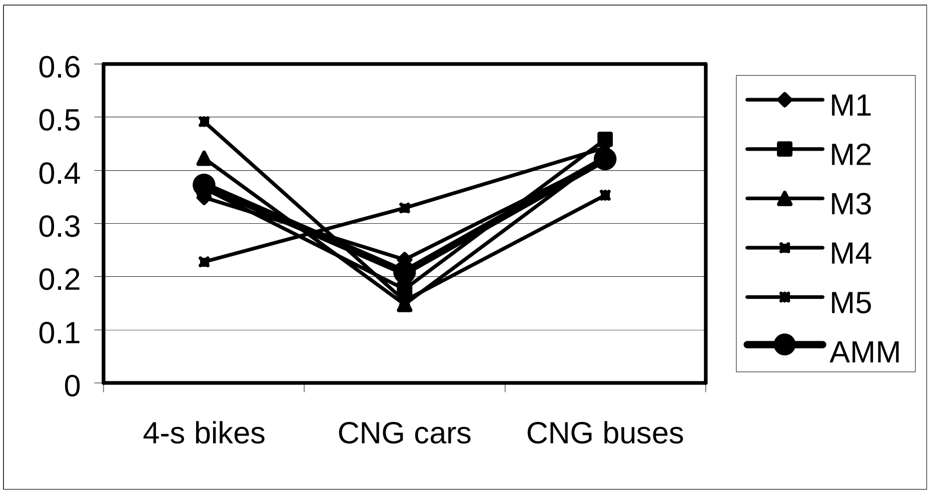

Fig. 6. Deviation of individual member priorities from group consensus based on AMM

All individual members of the groups followed similar trend in their priorities for the alternatives except the policy maker. The contradictory result from GMM could be due to the fact that member 4 (policy maker) rated A2 much higher and also with a considerable difference from the competing alternatives. This considerable difference lead to a rank reversal in GMM. Understandably policy makers have a stronger understanding and influence on transport sector. However, while aggregating individual priorities to get a group consensus, GMM failed to follow non-dictatorship axiom due to the overriding influence of the opinion of M4. This clearly demonstrates the failure of GMM to satisfy the non-dictatorship axiom as well.

When compared to other GA methodologies it is interesting to observe that weighted arithmetic mean method with varying weights showed more deviation from individual ranking compared to that of WAMM with equal weights. It may be due to the fact that the weights might have got more biased as the sample size is restricted to 7 (one person per category). Increased sample size might minimize this bias in weight derivation for actors. Weighted Arithmetic Mean Method with equal weights proved its potential in the group aggregation against GMM. Both WeAMM and WAMM satisfied pareto optimality and non-dictatorship axioms. Another interesting finding from this study is the arithmetic mean of individual priorities resulting in much lesser deviation from individual priorities of the group members. Member M4 could not significantly influence the group consensus in the case of AMM unlike the case with GMM.

Thus, this study demonstrates the correctness of using arithmetic mean methods in the group decision making and also demonstrates the lack of potential for GMM in this department.

6. Conclusions

In the present study, group aggregation methodology adopted in AHP was tested with a case study of Delhi transport system. It was observed that among all group aggregation methods, GMM showed a poor performance with contradicting results from the individual preferences. All other group aggregation methods viz. WeAMM, WAMM and AMM resulted in concurring results with individual member priorities. It was further demonstrated that WAMM (weighted arithmetic mean method) with equal weights for the actors resulted in a better aggregation of individual priorities. GMM failed to satisfy Pareto optimality and non-dictatorship axioms where as WeAMM and WAMM satisfied these most popular and well accepted social choice axioms. The following are few major findings and conclusions from this study.

- GMM, the most widely adopted GA method of AHP, failed to satisfy pareto optimality and non-dictatorship axioms raising questions on its validity
- WAMM with intrinsically derived weights was found doing better than GMM in assessing group priorities for alternative options
- Overall Priorities of alternatives using different GA methods viz. GMM, W_eAMM, WAMM, AMM demonstrated that W_eAMM is the most appropriate and efficient method to be applied in AHP for group aggregation.

- Comparative analysis between individual and group priorities demonstrated the deviations and arithmetic mean (AMM) of priorities by individual members of the group showed minimum deviation from the group consensus making it the most suitable and simple method to aggregate individual preferences to arrive at a group preference.
- To achieve energy efficiency and emission mitigation in Delhi transport system CNG bus got the top rank followed by 4-stroke 2-wheelers and CNG cars.

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