WP-2006-005

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October 2006

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Abstract

In a recent paper, Mishra and Subramanian (2006) propose a measure to explain group-differential which is sensitive to levels in the sense that a given hiatus at lower levels of failure (or higher levels of attainment) is considered worse off. This paper critically evaluates their method - refines their two axioms, adds an additional axiom of normalization and proposes an alternative which is more general. It proposes to reduce subjectivity when there is lower hiatus at lower levels of failure and also addresses scenarios when rank ordering of sub-groups will be reversed. Empirical illustration with infant mortality rate data for selected Indian states is also provided.

Key words: Difference-based, indicator of failure, level sensitivity, ratio-based.

JEL Code: D63

[1] The author thanks Udaya S. Mishra, Manoj Panda, S. Subramanian, M. H. Suryanarayana and students of 'Contemporary Issues in Human Development and Policy' (January-May 2006) at IGIDR for discussions and comments. Usual disclaimers apply. This paper is forthcoming in *Applied Economics Letters.*

On Measuring Group-differentials

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1. Introduction

In a recent paper, Mishra and Subramanian (2006) (hereafter, Mishra-Subramanian) propose a method of evaluating differences in the levels of certain socio-economic indicators. They suggest that if there is an indicator for some 'failure' that compares two sub-groups over two situations then the measure should exhibit some sensitivity to the levels. In line with the transfer-sensitivity property of poverty indices (Kakwani, 1993; Sen, 1976), they indicate that: "a given hiatus between two groups should acquire a greater salience the lower the level at which the hiatus arises." This is operationalized through two axioms, the difference-based and the ratio-based axioms, that are sensitive to the levels of the indicator. In this note we critically evaluate their method – refine their two axioms, add a third axiom of normalization and propose an alternative which is more general. Empirical illustration is provided with infant mortality rate data for selected Indian states.

2. Notations and Concepts

I_{js} indicates value of socio-economic indicator, $I \in (0,1)$; 0=no failure and 1=complete failure,[2] for the j^{th} group ($j=a,b$) in situation s ($s=A,B$). D_{rs} indicates the r^{th} differential

[2] If an indicator has the maximum of $n>1$ then it can be reduced to the (0,1) domain, indicators of attainment like literacy rate should be replaced with illiteracy rate, if we are discussing an attainment indicator like income then a maximum may be posited and the actual observations subtracted from this to obtain an indicator of failure.

measure in situation s with respect to I_{as} and I_{bs}. A differential measure, which implores a value judgement that a greater or equal hiatus at lower levels of failure should be considered as more serious, has been operationalized through two axioms.

The difference-based level sensitivity (DBLS) axiom: If I_{aA}-I_{bA}≥I_{aB}-I_{bB}=h; h>0, I_{aA}-I_{bA}=h+u; u≥0 (alternatively, if I_{bA}-I_{aA}≤I_{bB}-I_{aB}=h'; h'<0, I_{bA}-I_{aA}=h'+u'; u'≤0) and I_{bA}<I_{bB} then the DBLS axiom requires that D_{rA}>D_{rB}.

The ratio-based level sensitivity (RBLS) axiom: If $I_{aA}/I_{bA} \geq I_{aB}/I_{bB}=k$; $k\in(1,n]$, $I_{aA}/I_{bA}=kv$; $v\in[1,n/k]$ (alternatively, if $I_{bA}/I_{aA} \leq I_{bB}/I_{aB}=k'$; $k'\in(0,1)$, $I_{bA}/I_{aA}=k'v'$; $v'\in(0,1]$) and I_{bA}<I_{bB} then the RBLS axiom requires that D_{rA}>D_{rB}.

In the above two axioms the restriction on h>0 and u≥0 under DBLS and k>1 and v≥1 under RBLS (or, alternatively h'<0 and u'≤0 under DBLS and k'<1 and v'≤1 under RBLS) indicate that $I_{as}>I_{bs}\forall s$ and along with I_{bA}<I_{bB} it indicates that I_{aA}<I_{aB} whereas the restriction on the upper bound of k=n (or, lower bound of k'>0) indicates that $I_{bs}>0\forall s$. Now we propose a third axiom.

The normalization axiom: The differential measure should have a minimum and a maximum, $D_r\in[0,1]$ such that at the minimum it would indicate no group-differential and at the maximum it would indicate the highest group-differential.

There are four differential measures indicated in Mishra-Subramanian. Keeping the third axiom in mind and by assuming that $I_{as}>I_{bs}\forall s$ we reconstruct these four measures:

$$D_1=I_a-I_b \quad (1)$$

$$D_2=I_a^{\delta}-I_b^{\delta};\ 0<\delta<1 \quad (2)$$

$$D_3=1-I_b/I_a \quad (3)$$

$$D_4=1-I_b^{\alpha+1}/I_a^{\alpha};\ \alpha>0. \quad (4)$$

D_1 is the popularly used difference between two sub-groups. It satisfies the DBLS axiom in a weak sense, that is, if $I_{aA}-I_{bA}>I_{aB}-I_{bB}$ then $D_{1A}>D_{1B}$ and does not satisfy the RBLS axiom. D_2 is a modified version of D_1 that satisfies the DBLS axiom but it also does not satisfy the RBLS axiom. D_3 is another modification of D_1 and a normalized version of the popularly used ratio between two sub-groups. It satisfies the DBLS axiom and the RBLS axiom in a weak sense, that is, if $I_{bA}/I_{aA}<I_{bB}/I_{aB}$ then $D_{3A}>D_{3B}$. D_4 satisfies both the DBLS as well as RBLS axioms.[3]

3. An Extension

An extension of D_4 is proposed as,

$$D_5=1-I_b^{\alpha+\beta}/I_a^{\alpha};\ \alpha>0,\ \beta\geq 0. \quad (5)$$

If $\beta=0$ then DBLS is satisfied, but not RBLS – a special case of this is at $\alpha=1$ where $D_5=D_3$. In D_5, $\alpha>0$ indicates the similarity with D_4. In particular, if $\beta=1$ then $D_5=D_4$. If $\beta>0$ then D_5 satisfies the DBLS and the RBLS axioms.

For DBLS axiom not to hold and if $I_{aA}-I_{bA}\geq I_{aB}-I_{bB}=h$; $h>0$, $I_{aA}-I_{bA}=h+u$ and $u\geq 0$ then by substitution in D_5 we will have $(1-I_{bA}^{\alpha+\beta}/(I_{bA}+h+u)^{\alpha})\leq(1-I_{bB}^{\alpha+\beta}/(I_{bB}+h)^{\alpha})$ or $(I_{bA}^{\alpha+\beta}/(I_{bA}+h+u)^{\alpha})\geq(I_{bB}^{\alpha+\beta}/(I_{bB}+h)^{\alpha})$ or $(I_{bA}/(I_{bA}+h+u))^{\alpha})I_{bA}^{\beta}\geq(I_{bB}/(I_{bB}+h))^{\alpha})I_{bB}^{\beta}$; which

[3] Proofs for the axioms to hold for D_4 will be similar to the proofs for the extension proposed by us in D_5 and discussed in section 3.

however is not true because $I_{bA}<I_{bB}$. Similarly, for RBLS not to hold and if $I_{bA}/I_{aA}\leq I_{bB}/I_{aB}=k'$; $k'\in(0,1)$, $I_{bA}/I_{aA}=k'v'$; $v'\in(0,1]$ then by substitution in D_5 we will have $(1-I_{bA}^{\alpha+\beta}/I_{aA}^{\alpha})\leq(1-I_{bB}^{\alpha+\beta}/I_{aB}^{\alpha})$ and by manipulation we will have $I_{bA}^{\alpha+\beta}/I_{aA}^{\alpha}\geq I_{bB}^{\alpha+\beta}/I_{aB}^{\alpha}$ or $(k'v')^{\alpha}I_{bA}^{\beta}\geq(k')^{\alpha}I_{bB}^{\beta}$ which however is not true because $I_{bA}<I_{bB}$.

Using D_5, comparison between two situations is easy when $u\geq0$ or $v'\leq1$ - it is robust to $\beta>0$, however small it may be. Now, suppose we have a situation where $u<0$ and $v'>1$, but other conditions remain such that $I_{bA}<I_{bB}$, $h+u>0$ and $k'v'\in(0,1)$, then what should be the basis of our comparison. It will depend upon $\beta=f(I_{aA}, I_{bA}, I_{aB}, I_{bB}, \alpha)$, that is, if $\beta\gtreqless c=(\log_{(I_{bA}/I_{bB})}(I_{aA}/I_{aB})^{\alpha}-\alpha)$ then $D_{5A}\lesseqgtr D_{5B}$. One can always choose a higher value of β so that $D_{5A}>D_{5B}$. To reduce such possibilities, we propose a lower value of β.

Another caveat is in order for D_5. The restrictions that we put on $h>0$ or $h+u>0$ and the lower bound of $k'<1$ or $k'v'<1$ would at times lead to a loss of social meaning. For instance, if in one situation one sub-group has a lower level and in another situation the other sub-group has a lower level, $I_{aA}<I_{aB}$ & $I_{bA}>I_{bB}$; which can also mean that $I_{aA}<I_{bA}$ & $I_{aB}>I_{bB}$. It will lead to $h<0$, but $h+u>0$ (alternatively, $k'>1$, but $k'v'<1$). In such situations, D_5 may not satisfy the DBLS and RBLS axioms.

One can still compare, by ordering the sub-groups in each situation from high to low and by computing overall group-differentials by replacing the sub-groups a and b with $\max(I_a,I_b)$ and $\min(I_a,I_b)$ respectively. In such scenarios, there is change in social dynamics that shifts one group from an advantageous position to a disadvantageous position. If the base scenario is $I_a>I_b$ then in those scenarios where it is reversed, $I_a<I_b$, we propose to indicate the value with a negative sign, $-D_r$, so as to enable comparison. The group-differential in both the situations being the absolute value of the measure, $|-D_r|=|D_r|=D_r$.

If either $I_a=0$ or $I_b=0$ then the level sensitive measures of D_4 and D_5 will be -1 or $+1$ respectively – the absolute value indicating highest differential. It means that if one sub-group has no failure and the other has some failure the level sensitive differential measures indicate the maximum possible hiatus.

If $I_a=I_b=1$ then $D_r=0$ in all the five proposed measures. However, if $0<I_a=I_b<1$ then the measures of D_1, D_2, and D_3 will be zero, but the level sensitive measures of D_4 and D_5 will give us a value, which is indifferent to sub-group ordering – we will indicate such a value as $\pm D_r$. More importantly, this value increases as I decreases. It follows that as $I_a=I_b=I\to 0$, $D_4\to 1$ and $D_5\to 1$. This sounds strange because in a situation when both sub-groups have no failure our proposed measure indicates the highest differential. This is inherent in our level sensitive measures which give a greater weight to lower values of I. The merit of the proposed measure of group-differential satisfying the two axioms (particularly, RBLS axiom) has to be taken with this pinch of salt.

4. Empirical Illustration

The empirical exercise uses infant mortality rate (IMR) data from selected Indian states. The results are given in Table 1. The first case analyses female-to-male gender gap in the states of Karnataka and Orissa in 2003. D_1 indicates equal difference, I_{aA}-

$I_{bA}=I_{aB}-I_{bB}$. All the other measures satisfy the DBLS axiom and indicate a higher gender gap in Karnataka. The second case analyses rural-to-urban gap for Assam in 1990 and 2003. D_1 and D_2 indicate greater gap in 1990, D_3 indicates equal ratio, $I_{bA}/I_{aA}=I_{bB}/I_{aB}$, and the remaining two measures satisfy the RBLS axiom indicating a greater gap in 2003. In the third case the male-to-female gender gap is lower in urban Andhra Pradesh when compared with urban Rajasthan in the sense that $I_{aA}/I_{bA}<I_{aB}/I_{bB}$ (or $I_{bA}/I_{aA}>I_{bB}/I_{aB}$). This is reflected in our first three differential measures. This need not be the case in our level sensitive measures. A relatively higher value of β=1 makes Andhra Pradesh worse off than Rajasthan in D_4 whereas a lower value of β=0.001 does not do this in D_5.

Table 1
Comparing Group-differential under Various Measures Using
Infant Mortality Rate Data from Indian States

		I_a	I_b	D_1	D_2	D_3	D_4	D_5
Case 1	Karnataka, 2003	0.052	0.051	0.0010	0.0192	0.0022	0.9500	0.0221
$I_{aA}-I_{bA}=I_{aB}-I_{bB}$	Orissa, 2003	0.083	0.082	0.0010	0.0120	0.0017	0.9190	0.0145
Case 2	Assam, 2003	0.070	0.035	0.0350	0.5000	0.0775	0.9825	0.5017
$I_{bA}/I_{aA}=I_{bB}/I_{aB}$	Assam, 1990	0.078	0.039	0.0390	0.5000	0.0818	0.9805	0.5016
Case 3	AP, Urban 2003	0.036	0.030	0.0060	0.1667	0.0165	0.9750	0.1696
$I_{bA}/I_{aA}>I_{bB}/I_{aB}$	Rajasthan, Urban 2003	0.058	0.047	0.0110	0.1897	0.0240	0.9619	0.1921
Case 4	Punjab, Rural 2003	0.057	0.049	0.0080	0.1404	0.0174	0.9579	0.1429
$I_{aA}>I_{bA}$, $I_{aB}<I_{bB}$	HP, Rural 2003	0.039	0.047	-0.0080	-0.1702	-0.0193	-0.9676	-0.1729
Case 5	Kerala, Rural 2003	0.012	0.012	0	0	0	±0.9880	±0.0044
$I_{aA}=I_{bA}<I_{aB}=I_{bB}$	WB, Rural 2003	0.048	0.048	0	0	0	±0.9520	±0.0030

Notes: I_a and I_b denote infant mortality converted to the 0-1 range for sub-groups *a* and *b* respectively; D_1, D_2, D_3, D_4 and D_5 denote the five differential measures discussed in the text; D_3 has been computed for δ=0.5; D_4 has been computed for α=1; D_4 is a special case of D_5 when α=1 and β=1, but we have also independently computed D_5 with α=1 and β=0.001. In all cases, situations *A* and *B* are indicated in the first and second rows respectively. Sub-groups *a* and *b* refer to female and male respectively in cases 1, 4 and 5, rural and urban respectively in case 2 and male and female respectively in case 3. In case 4, sub-group ordering of Punjab is taken as the base and the reverse ordering in HP is indicated through a negative sign. In case 5, ± denotes that the values of D_4 and D_5 are indifferent to sub-group ordering. AP, HP and WB refer to Andhra Pradesh, Himachal Pradesh and West Bengal respectively.

Sources: *Sample Registration System Statistical Report 2003*, Report No. 2 of 2005, Registrar General, India, New Delhi. *Vital Statistics of India 1990 Based on the Civil Registration System*, Office of the Registrar General, India, Ministry of Home Affairs, New Delhi.

In the fourth case the sub-group ordering is reversed – Punjab has greater female infant mortality rate, $I_{aA}>I_{bA}$, whereas Himachal Pradesh has lower female infant mortality rate, $I_{aB}<I_{bB}$. We calculate female-to-male gender gap for Punjab and male-to-female gender gap for Himachal Pradesh and considering the first to be the base scenario we indicate the latter with a negative sign. The absolute difference is equal, as indicated in the D_1 measure, but for all other measures the absolute values are higher in Himachal Pradesh where levels are lower. Females are a sturdier population and lower female infant mortality is only natural whereas a higher female infant mortality rate indicates the presence of social and other forces leading to this gap. Thus, negative values for Himachal Pradesh can also be interpreted as one where a gap that is advantageous to females should be considered lower than a gap that is advantageous to males.

The fifth case analyses scenarios where both the sub-groups have equal failure, $0<I_a=I_b<1$. In such situations $D_1=D_2=D_3=0$ whereas D_4 and D_5 give us positive values though they are indifferent to sub-group ordering. More importantly, this value increases as the level of failure decreases. It is greater for rural Kerala than that for rural West Bengal.

5. Concluding Remarks

This paper discusses about measures of group-differentials. In particular, it discusses the merits and demerits of level sensitive measures where greater or equal hiatus at lower levels of failure (or higher levels of attainment) is considered worse off. It proposes to reduce subjectivity when there is lower hiatus at lower levels of failure and also addresses scenarios when rank ordering of sub-groups will be reversed. Empirical illustration with infant mortality rate data for selected Indian states is also provided. The proposed measure can be used to compare group-differential across situations. One possible application in the current context is to evaluate the progress of Millennium Development Goals in terms of group-differentials. Extending the measure to multiple groups is another challenge. Even more interesting would be to introduce level sensitivity to various measures of inequality.

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