THE SUB – OPTIMALITY OF THE OPPORTUNISTIC APPROACH
TO DISINFLATION‡

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Abstract

One approach to achieving price stability is to undertake a deliberate path to an ultimate goal of low inflation - deliberate disinflation. In contrast an opportunistic strategy for disinflation has gained credence in recent years. We compare the ability of the two approaches to achieve macroeconomic stability and conclude that the opportunistic approach is sub-optimal when a commitment mechanism is in place. We show that a nonlinear effect of the shock on the position of the Phillips curve trade-off along with adaptive expectations yields an opportunistic inflation response. However, such nonlinear shift effects have no theoretical underpinning implying that the theory for opportunism is weak.

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I Introduction

Central banks in different countries have adopted different strategies for achieving price stability. During the 1990s several countries have introduced explicit inflation targets.\(^1\) One approach to price stability is to take a *deliberate* path to an ultimate goal of low inflation. In contrast, an *opportunistic* disinflation strategy specifies both an *interim* as well as a *long-run* goal for price stability. Proponents of this approach hold that when inflation is moderate but still above the central bank’s long-run inflation objective, policymakers should not take deliberate anti-inflation action, but rather should wait for favourable supply shocks and unforeseen recessions to deliver the desired reduction in inflation. This strategy has gained credence among some prominent central bankers and academics in recent years.\(^2\)

Commenting on the FOMC’s strategy in the 1990s, Alan Blinder (1997) writes

> “Under certain circumstances, the optimal disinflation strategy is asymmetric in the following specific way: you guard vigorously against any rise in inflation, but wait patiently for the next favourable inflation shock to bring inflation down. The opportunistic strategy makes the time needed to approach the ultimate inflation target a random variable. When I was the Vice Chairman of the Fed, I often put it this way: the United States is “one recession away” from price stability.”

\(^1\)For an indepth analysis see Bernanke et al. (1999).
\(^2\)For academic work on this topic see Orphanides and Wilcox (1996), Orphanides et al. (1997) and Bomfim and Rudebusch (2000).
Explaining the opportunistic disinflation strategy Governor Laurence Meyer (1996) notes “Under this strategy, once inflation becomes modest, as today, Federal Reserve policy in the near term focuses on sustaining trend growth at full employment at the prevailing inflation rate. At this point the short-run priorities are twofold: sustaining the expansion and preventing an acceleration of inflation. This is, nevertheless, a strategy for disinflation because it takes advantage of the opportunity of inevitable recession and potential positive supply shocks to ratchet down inflation over time.”

In this paper we compare the ability of the two approaches to achieve macroeconomic stability (measured in terms of inflation and output variability) when a policymaker commits to a particular strategy. We assume commitment argument on the assumption that the central bank has full political backing for the policy of inflation control and faces no pressure to use monetary policy to raise the flexible long-run employment rate; this is the usual framework within which opportunism is discussed. The key difference between a deliberate and an opportunistic policymaker is in the reaction to deviations of inflation from target. First, while the deliberate policymaker reacts to the gap between actual inflation from long-run target, the opportunistic policymaker reacts to the gap between actual inflation and an interim target. Second, while the deliberate policymaker responds to the inflation gap in a linear manner, the opportunistic policymaker’s reaction to the gap between actual inflation and the interim target is nonlinear. We demonstrate that such asymmetries
result in higher inflation (and output) variability under commitment in spite of zero inflation bias. An interesting question is what considerations could motivate the policymaker to adopt an objective function with these characteristics; hitherto such models have been studied by assuming that expectations are adaptive. Specifically we show that a nonlinear effect of the shock on the position of the Phillips curve trade-off along with adaptive expectations provides justification for an opportunistic strategy.

The rest of the paper is organised as follows. In section II we review the usual deliberate approach to disinflation under commitment. This is followed by the derivation of the optimal inflation response under commitment when a policymaker is opportunistic in section III, which includes a comparison of the two strategies. In section IV we explore the economic rationale for opportunism and conclude that it is motivated by political economy considerations. Section V concludes the paper.

II Deliberate strategy under commitment

There is by now widespread agreement among central bankers and academics alike that inflation targeting in practice is ‘flexible’ inflation targeting. The central bank’s objective is not only to stabilize inflation around an exogenously specified target,

\footnote{Bomfim and Rudebusch (2000)} explore the role of imperfect credibility and opportunism in a model with adaptive expectations and conclude that opportunism is sub-optimal.
but also to put some weight on stabilizing the output gap.\textsuperscript{4} There is also general agreement that inflation-targeting central banks do not have overambitious output targets. Hence, discretionary optimization \textit{à la} Kydland and Prescott (1977) or Barro and Gordon (1983) (KPBG hereafter), does not result in average inflation bias (see Blinder, 1997, 1998).

(i) \textit{Deliberate strategy under commitment}


The short-run Phillips curve is

\[ y_t = \rho y_{t-1} + \alpha (\pi_t - \pi_t^e) + \varepsilon_t, \]  \hfill (2.1)

where \( y_t \) is the output gap in period \( t \), \( \alpha \) and \( \rho \) are constants (\( \alpha > 0 \) and \( 0 < \rho < 1 \)), \( \pi_t \) is the inflation rate, \( \pi_t^e \) denotes expectations conditional upon information available in period \( t - 1 \), and \( \varepsilon_t \) is iid error with mean zero and variance \( \sigma^2 \). The private sector has rational expectations; that is,

\[ \pi_t^e = E_{t-1} \pi_t, \]  \hfill (2.2)

Now suppose that there is a commitment mechanism, so that the central bank

\textsuperscript{4}A positive weight on output gap is generally considered to be consistent with the mandate of many central banks to not only maintain price stability but also facilitate economic growth over time.
can commit to the optimal rule. Under commitment, the optimal rule under inflation targeting is

\[ \pi_t = \pi^*_t + b\varepsilon_t, \quad (2.3) \]

where, under commitment, inflation is independent of the lagged output gap and only depends on the new information that has arrived after the private sector formed its expectations. When the central bank is committed to a state-contingent rule in conducting monetary policy, this implies that the monetary authority internalizes the impact of its decision rule on the expectations of the private sector. In other words, the monetary authority takes into account how its actions affect the private sector’s expectations. It does this by minimizing its loss function with respect to the private sector’s expectations of the inflation rate under the explicit constraint that these expectations are formed rationally.

Thus, (2.1), (2.2) and (2.3) represent the constraints facing the central bank. The central bank’s objective under deliberate disinflation strategy is to stabilize inflation around a given (long-run) inflation target, \( \pi^* \), as well as stabilizing the output gap around an output gap target, \( y^* = 0 \). This can be represented by an intertemporal loss function for the central bank given by

\[ E_t \left[ \sum_{\tau=t}^{\infty} \beta^{\tau-t} L_\tau \right], \quad (2.4) \]
with the period loss function

\[ L_t = \frac{1}{2} \left[ (\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 \right], \quad (2.5) \]

where \( \lambda > 0 \) is the relative weight on output-gap stabilization. The central bank is, for simplicity, assumed to have perfect control over the inflation rate \( \pi_t \). It sets the inflation rate in each period after having observed the current supply shock \( \varepsilon_t \). This is a dynamic programming problem with one state variable, \( y_{t-1} \), and two control variables, \( \pi_t \) and \( \pi_t^e \), and where \( \beta \) is the discount factor.\(^5\) The solution can be obtained by solving the following equation involving the value function \( V(y_t) \). Thus, the decision problem of the central bank can be expressed as

\[ V(y_{t-1}) = E_{t-1} \min_{\pi_t^e, \pi_t} \left\{ \frac{1}{2} \left[ (\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 \right] + \beta V(y_t) \right\}, \quad (2.6) \]

where the minimization in period \( t \) is subject to (2.1)-(2.3). When the central bank is committed to a state-contingent rule in conducting monetary policy, it implies that the monetary authority internalises the impact of its decision rule on the expectations of the private sector. In other words, the monetary authority takes into account how its actions affect the private sector’s expectations. For the linear-quadratic problem such as ours, \( V(y_t) \) must also be quadratic. Thus, the indirect loss function can be

\(^5\)Note that if there is no output persistence, the problem of minimizing the intertemporal loss function Eq. (2.4) is equivalent to the static problem of minimizing the expected period loss function Eq. (2.5).
written as
\[
V(y_{t-1}) = \gamma_0 + \gamma_1 y_{t-1} + \frac{1}{2} \gamma_2 y_{t-1}^2,
\]  
(2.7)

so that \(V'(y_{t-1}) = \gamma_1 + \gamma_2 y_{t-1}\) and \(\gamma_i's\) are the undetermined coefficients. Using this condition together with Eqs. (2.1)-(2.3), we obtain two first-order conditions from Eq. (2.6) with respect to \(\pi_t^*\) and \(\pi_t\), respectively:

\[
E_{t-1}\pi_t = \pi^* 
\]  
(2.8)

\[
\mu = -(\pi_t - \pi^*) - \lambda \alpha (y_t - y^*) - \alpha \beta \left[\gamma_1 + \gamma_2 (y_t - y^*)\right] 
\]  
(2.9)

where \(\mu\) is the Lagrangian multiplier on the joint constraint (2.2) and (2.3): \(\mu (\pi_t - E_{t-1}\pi_t + b\varepsilon_t)\). Taking expectations of (2.9) and substituting (2.8) for \(E_{t-1}\pi_t\) implies that

\[
\mu = -\lambda \alpha (\rho y_{t-1} - y^*) - \alpha \beta \left[\gamma_1 + \gamma_2 (\rho y_{t-1} - y^*)\right] 
\]  
(2.10)

Substituting (2.10) in (2.9) for \(\mu\) yields:

\[
\pi_t = \pi^* - \frac{\alpha (\beta \gamma_2 + \lambda)}{1 + \alpha^2 (\beta \gamma_2 + \lambda)} \varepsilon_t 
\]  
(2.11)

Eq. (2.11) is the optimal feedback rule for inflation under commitment expressed as a function of the parameters of the model and the coefficient, \(\gamma_2\), which can be easily derived by making use of the Envelope theorem. Differentiating Eq. (2.6) w.r.t \(y_{t-1}\) yields:

\[
V'(y_{t-1}) = \gamma_1 + \gamma_2 y_{t-1} = \rho \lambda (\rho y_{t-1} - y^*) + \beta \rho [\gamma_1 + \gamma_2 (\rho y_{t-1} - y^*)] 
\]  
(2.12)
Collecting terms in $\gamma_2$ yields:

$$\gamma_2 = \frac{\lambda \rho^2}{1 - \beta \rho^2}$$  \hspace{1cm} (2.13)

Therefore, the solution for inflation and output gap under a deliberate strategy can be expressed as:

$$\pi_t = \pi^* - \left[ \frac{\alpha \lambda}{1 - \beta \rho^2 + \alpha^2 \lambda} \right] \varepsilon_t$$  \hspace{1cm} (2.14)

$$y_t = \rho y_{t-1} + \left[ \frac{1 - \beta \rho^2}{1 - \beta \rho^2 + \alpha^2 \lambda} \right] \varepsilon_t$$  \hspace{1cm} (2.15)

where the average inflation bias, $E(\pi_t) - \pi^* = 0$ i.e., there is no average inflation bias with a deliberate strategy under commitment. The unconditional variability of both output and inflation will be proportional to the variance of the supply shock.

III Opportunistic strategy under commitment

In contrast to a deliberate policymaker the opportunistic policymaker reacts to the gap between actual inflation and an interim target. He guards against any incipient rise in the interim target for inflation ($\pi^*_t$), but waits for the next favourable inflation shock to lower the interim target, rather than seeking to actively lower the interim target towards the long-run target ($\pi^*$). This difference in policy responsiveness invariably suggests a nonlinearity in the policy response function. In this section we extend the standard KPBG analysis by assuming that the interim target for inflation depends on the realisation of supply shocks. Thus, the opportunistic policymaker is
assumed to minimise

\[ V(y_{t-1}) = E_{t-1} \min_{\pi^*_t, \pi_t} \left\{ \frac{1}{2} \left[ (\pi_t - \pi^*_t)^2 + \lambda (y_t - y^*)^2 \right] + \beta V(y_t) \right\}, \] (3.1)

where \( \pi^*_t \) is the interim target for inflation and \( V(y_t) \) is defined in Eq. (2.7). In addition the model includes an equation describing the determination of the intermediate target as a function of the underlying supply shock and a weighted average of past inflation and the long-run target for inflation. In other words we assume that policymakers incentive to deflate is a nonlinear function of the underlying supply shock i.e.,

\[ \Delta \pi^*_t = -\delta (e^{\gamma \pi_{t-1}}) - \phi (\pi^*_t - \pi^*) + \delta \left( e^{\frac{-\gamma^2 \pi^2}{2}} \right) \]

or

\[ \pi^*_t = \pi^* - \delta \sum_{i=0}^{\infty} (1 - \phi)^i e^{\gamma \pi_{t-1-i}} + \frac{\delta}{\phi} \left( e^{\frac{-\gamma^2 \pi^2}{2}} \right) \] (3.2)

where \( \delta, \phi, \gamma > 0 \). Thus, the intermediate target always lies between the inherited inflation target \( (\pi^*_{t-1}) \) and the long-run target \( (\pi^*) \).\(^6\) Note that the opportunistic central banker reacts asymmetrically to supply shocks. Figure 1 plots Eq. (3.2) for \( \delta = 0.1 \) and for \( \gamma = 1.5 \) (assuming that \( \phi = 0 \)). The x-axis plots both positive and negative deviations of supply shocks while the y-axis plots the implied change in the

\(^6\)The key feature of the interim target is that it exhibits path dependence i.e., allows the policymaker to react differently to a given level of inflation depending on the prior history of inflation itself (see Orphanides and Wilcox, 1996).
interim target for inflation. It is clear from the figure that when there is a positive supply shock the interim target for inflation is adjusted downwards while it stays put when we have negative supply shocks. Also note from Eq. (3.2) that in the long-run the interim target converges to the long-run target i.e., when supply shocks are zero, \( \pi_t^T = \pi^* \).

![Figure 1: Change in the Interim Target for Inflation](image)

Using (3.2) together with Eqs. (2.1)-(2.3), we obtain two first-order conditions under commitment from Eq. (3.1) with respect to \( \pi_t^e \) and \( \pi_t \), respectively:

\[
E_{t-1}\pi_t = E_{t-1}\pi_t^T = \pi^* - \delta \sum_{i=0}^{\infty} (1 - \phi)^i \phi^i e^{\kappa t-1-i} + \frac{\delta}{\phi} \left( e^{\frac{z^2\sigma^2}{2}} \right) \quad (3.3)
\]

\[
\mu_1 = -\left( \pi_t - \pi_t^T \right) - \lambda \alpha (y_t - y^*) - \alpha \beta [\gamma_1 + \gamma_2 (y_t - y^*)] \quad (3.4)
\]

where \( \mu_1 \) is the Lagrangian multiplier. Taking expectations of (3.4) and substi-

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Note that the constant term \( \delta \left( e^{\frac{z^2\sigma^2}{2}} \right) \) in Eq. (3.2) just shifts Figure 1 upwards.
tuting (3.3) for $E_{t-1} \pi_t$ implies that

$$
\mu_1 = -\lambda \alpha (\rho y_{t-1} - y^*) - \alpha \beta [\gamma_1 + \gamma_2 (\rho y_{t-1} - y^*)]
$$

(3.5)

Substituting (3.5) in (3.4) for $\mu_1$ yields:

$$
\pi_t = \phi \pi^* + (1 - \phi) \pi^T_{t-1} - \delta (e^{\gamma_1 t-1}) + \delta \left( e^\frac{\gamma_2^2 \sigma^2}{2} \right) - \left[ \frac{\alpha (\beta \gamma_2 + \lambda)}{1 + \alpha^2 (\beta \gamma_2 + \lambda)} \right] \varepsilon_t
$$

(3.6)

where as before $\gamma_2 = \frac{\lambda \sigma^2}{1 - \beta \rho^2}$ is derived by exploiting the Envelope theorem. Eq. (3.6) is the optimal feedback rule for a opportunistic central banker under commitment expressed as a function of the parameters of the model and the coefficient, $\gamma_2$. Note that the average inflation bias under an opportunistic strategy is also zero,$^8$

$$
E(\pi_t - \pi^*) = -\delta E\sum_{i=0}^{\infty} (1 - \phi)^i e^{\gamma_1 t-1 - i} + \delta \frac{\gamma_2}{\phi} \left( e^\frac{\gamma_2^2 \sigma^2}{2} \right) = 0
$$

(3.7)

where $\pi_t = E_{t-1} \pi_t + b \varepsilon_t$. To understand why, recall that the objective function of an opportunistic policymaker Eq. (3.1) is quadratic in spite of reacting asymmetrically to underlying shocks. In other words certainty equivalence still holds. However note that the unconditional variance of inflation (and output) is higher under opportunism and as a result is sub-optimal from a welfare point of view, as evidenced by Eq. (3.8).$^9$

$$
E(\pi_t - \pi^*)^2 = \delta^2 e^{\gamma_1^2 \sigma^2} \left( \frac{\phi e^{\gamma_2^2 \sigma^2} - (2 - \phi)}{\phi^2 (2 - \phi)} \right) + b^2 \sigma^2
$$

(3.8)

Note that the variance depends upon the asymmetry parameter $\gamma$. Clearly inflation variance is higher as one increases $\gamma$ i.e., more opportunistic the policymaker is the

$^8$See appendix A for derivation of the nonlinear expectation.

$^9$See appendix B for derivation of the unconditional variance of inflation.
higher the inflation variance. These results arise in spite of policies being fully credible and policymakers targeting potential output. The intuition behind this results is that under a deliberate strategy inflationary expectations are anchored by $\pi^*$. Whereas under opportunism inflationary expectations are random i.e., they vary with supply shocks. Consequently, actual inflation is more variable under opportunism. Thus, if the central bank’s loss function is quadratic in inflation and output deviations but responds asymmetrically to supply shocks, the opportunistic approach to disinflation is not optimal. On the contrary, the policymaker should in that circumstance pursue the objective of price stability period by period, regardless of the underlying shock as long as inflation is above its long-run target. In light of the fact that this policy is sub-optimal, it is important to understand why policymakers would pursue it.

IV A Rationale for the opportunistic approach to disinflation\textsuperscript{10}

The most important unresolved issue related to opportunism concerns the economic rationale for an objective function such as Eqs. (3.1)-(3.2). In other words

\textsuperscript{10}Indeed note that asymmetric preferences will deliver asymmetric response. But is there any justification for such preferences? Rotemberg and Woodford (1999) argue that a quadratic loss function is an approximation of the true social welfare function. If the true social welfare function is quadratic there is no obvious reason why policymaker’s preference should be asymmetric. Also in terms of observed risks is not deflation a concern as well as inflation; unemployment as well as overheating? Hence in our analysis we adopt the quadratic loss function, appealing to the justification offered in Rotemberg and Woodford.
what considerations could motivate the policymaker to adopt an objective function with these characteristics? Two arguments are cited in the literature as a justification for opportunism. The first emphasises the importance of expectations in the inflation process. That is, a credible disinflation policy will translate more quickly into lower inflation expectations and hence requires a smaller sacrifice of output. However, when expectations are adaptive, then inflation can be reduced with a higher transitional cost in terms of lost output, in parts because the necessary adjustments to nominal contracts take more time. Hence, authorities wait for favourable supply shocks to bring inflation down rather than engineer a downturn by pushing-up interest rates. Orphanides et al. (1997) among others use this argument to justify an opportunistic strategy. Second, a nonlinear Phillips curve provides a partial rationale for opportunism even when policymaker’s preferences are quadratic (Orphanides and Wilcox (2000)). The point is that with a nonlinear Phillips curve the sacrifice ratio is not independent of the size of an intended change in inflation i.e., the slope of the Phillips curve is inversely related to the sacrifice ratio. This suggests that the design of monetary policy may exhibit opportunism when inflation is relatively low. In what follows we investigate these arguments in turn and examine whether they rationalise opportunism.

(i) *Adaptive expectations and optimal policy rule*

To examine this we consider the following stylised model. We assume that the
central bank minimises Eq. (2.5) subject to

\[ y_t = y^* + \alpha (\pi_t - \pi_t^e) + \varepsilon_t, \quad (4.1) \]

where \( y^* \) is potential output and Eq. (4.1) represents the constraints facing the central bank as before. In addition we assume that expectations of inflation rate are adaptive and are determined by

\[ \pi_t^e - \pi_{t-1}^e = \alpha \left( \pi_{t-1} - \pi_{t-1}^e \right), \quad (4.2) \]

where \( 0 < \alpha < 1 \). Using Eq. (4.2) together with Eq. (4.1), we obtain the first-order condition from Eq. (2.5) with respect to \( \pi_t \):

\[ \pi_t = \pi_t^r - \left( \frac{\alpha \lambda}{1 + \alpha^2 \lambda} \right) \varepsilon_t, \quad (4.3) \]

where \( \pi_t^r = \left( \frac{1}{1 + \alpha^2 \lambda} \right) \pi^* + \left( \frac{a \alpha^2 \lambda}{1 + \alpha^2 \lambda} \right) \sum_{i=0}^{\infty} (1 - a)^i \pi_{t-1-i} \)

Note that the interim target is calculated as a weighted average of the long-run target and the inherited rate of inflation. The latter is simply taken to be a backward-looking moving average of actual inflation. Note that adaptive expectations does introduce an interim target and so goes part of the way to the opportunistic model. However, it does not rationalise an asymmetric response to shocks which is an important element in the model. What this suggests is that it is necessary to entertain alternatives to the linear-quadratic paradigm in order to rationalise opportunism.\(^{11}\)

\(^{11}\)A nonlinear Phillips curve with quadratic central bank preference does not yield a closed-form
(ii) Asymmetric effects of the shock on the Phillips curve

(a) The finite horizon case

In this section we assume that the Phillips curve is linear but the effect of the shock itself on the position of the trade-off is nonlinear. To examine the implication of this modification we propose the following functional form for the Phillips curve:

\[ y_t = y^* + \alpha (\pi_t - \pi_t^e) + \left( e^{bu_t} - 1 \right), \]  \hspace{1cm} (4.4)

where \( \alpha \) and \( b \) are positive constants and \( u_t \) is a conditionally normal error with mean zero and variance \( \sigma_u^2 \). In Eq. (4.4) output is assumed to respond asymmetrically to supply disturbances. In addition we assume that expectations of inflation rate are adaptive and are determined by Eq. (4.2) i.e.,

\[ \pi_t^e = \frac{aL\pi_t}{1 - (1 - a)L} = \frac{a\pi_{t-1}}{1 - (1 - a)L}, \]  \hspace{1cm} (4.5)

where \( L \) is the lag operator. The policymaker’s preference is given by the period loss function

\[ L_t = \frac{1}{2} \left[ (\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 \right] + \tau (\pi_t - \pi^*) \]  \hspace{1cm} (4.6)

where \( \lambda > 0 \) is the relative weight on output-gap stabilization and ‘\( \tau \)’ is the constant parameter of a Walsh (1995) inflation contract which is designed to eliminate solution for inflation (see Orphanides and Wieland (2000)). They show that a nonlinear Phillips curve provides a partial rationale for inflation zone targeting i.e., policy characterised by a zone of inaction for small deviations of inflation from target. However, such inaction applies equally to both positive as well as negative deviations and hence is not the same thing as opportunism.
the bias that comes from the inability to commit under adaptive expectations. Using
Eq. (4.4) we obtain the first-order conditions from Eq. (4.6) with respect to \( \pi_t \):

\[
\pi_t \left( 1 + \alpha^2 \lambda \right) = \pi^* + \alpha^2 \lambda \pi_t^e - \alpha \lambda \left( e^{bu_t} - 1 \right) - \tau \tag{4.7}
\]

Eq. (4.7) defines the first order condition for the optimal policy rule for inflation
under discretion. Substituting Eq.(4.5) for \( \pi_t^e \) in Eq.(4.7) yields:

\[
\pi_t = \frac{\pi^*}{1 + \alpha^2 \lambda} + \frac{aa^2 \lambda}{(1 + \alpha^2 \lambda) (1 - (1 - a)L)} \pi_{t-1} - \frac{\alpha \lambda \left( e^{bu_t} - 1 \right)}{1 + \alpha^2 \lambda} - \frac{\tau}{1 + \alpha^2 \lambda} \tag{4.8}
\]

By continuous forward substitution we have;

\[
\pi_t = \pi^* - \tau - \left( \frac{\alpha \lambda}{1 + \alpha^2 \lambda} \right) \sum_{i=0}^{\infty} \left( \frac{aa^2 \lambda}{(1 + \alpha^2 \lambda) (1 - (1 - a)L)} \right)^i \left( e^{bu_{t-i}} - 1 \right) \tag{4.9}
\]

where Eq. (4.9) (which is similar to Eq. (3.2)) defines the optimal inflation
response when the effect of the shock on the position of the Phillips curve trade-off
is nonlinear.\(^{12}\)

(b) The infinite horizon case

Suppose we add persistence to the Phillips curve Eq. (4.4) above i.e.,

\[
y_t = \rho y_{t-1} + \alpha \left( \pi_t - \pi_t^e \right) + \left( e^{bu_t} - 1 \right) , \tag{5}
\]

and assume that expectations are adaptive as in Eq. (4.5) then the problem in Eq.
(4.6) is a dynamic programming problem with one state variable, \( y_{t-1} \), and one control

\(^{12}\)From Eq. (4.9) we observe that to remove the inflation bias through using Walsh contract
\( \tau = \alpha \lambda \left( 1 - e^{\frac{\sigma^2}{\alpha^2}} \right) \).
variables, \( \pi_t \). Thus, the decision problem of the central bank can be expressed as

\[
V (y_{t-1}) = E_{t-1} \min_{\pi_t} \left\{ \frac{1}{2} \left( (\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 \right) + \beta V (y_t) \right\},
\]

(5.1)

where the minimization in period \( t \) is subject to (4.5) and (5). Because of the inability to commit under adaptive expectations the central bank in this case does not internalise the effect of its decisions on inflation expectations. The indirect loss function for this problem is the same as Eq. (2.7) above. The first-order condition from Eq. (5.1) with respect to \( \pi_t \), yields:

\[
\pi_t = \pi^* - \lambda \alpha (y_t - y^*) - \alpha \beta [\gamma_1 + \gamma_2 (y_t - y^*)]
\]

(5.2)

Substituting Eq. (4.5) for \( \pi_t^* \) and Eq. (5) for \( y_t \) in Eq. (5.2) yields:

\[
\pi_t = \pi^* + \frac{\alpha (\beta \gamma_2 + \lambda)}{1 + \alpha^2 (\beta \gamma_2 + \lambda)} - \frac{\alpha (\beta \gamma_2 + \lambda)}{1 + \alpha^2 (\beta \gamma_2 + \lambda)} \left( \rho y_{t-1} + \left( e^{b_{ut}} - 1 \right) \right) \\
+ \frac{\alpha (\beta \gamma_2 + \lambda) y^* - \beta \gamma_1 \alpha}{1 + \alpha^2 (\beta \gamma_2 + \lambda)} + k \pi_{t-1}
\]

(5.3)

where \( k = \left( \frac{\alpha}{1 - (1 - \alpha) \rho} \right) \left( \frac{\alpha^2 (\beta \gamma_2 + \lambda)}{1 + \alpha^2 (\beta \gamma_2 + \lambda)} \right) \). By continuous forward substitution we have:

\[
\pi_t = \pi^* + \alpha (\beta \gamma_2 + \lambda) y^* - \beta \gamma_1 \alpha \\
- \left( \frac{\alpha (\beta \gamma_2 + \lambda)}{1 + \alpha^2 (\beta \gamma_2 + \lambda)} \right) \left( \rho \sum_{i=0}^{\infty} k^i y_{t-1-i} + \sum_{i=0}^{\infty} k^i \left( e^{b_{ut-i}} - 1 \right) \right) 
\]

(5.4)

where \( \gamma_1 = -\frac{\lambda \rho}{1 - \beta \rho} y^* \) and \( \gamma_2 = \frac{\lambda \beta}{1 - \beta \rho} \) by making use of the Envelope theorem. Eq. (5.4) is the optimal feedback rule for inflation under discretion when the effect of the shock on the position of the Phillips curve trade-off is nonlinear. Note that
in Eq. (4.6) the inflation bias, in addition depends on past output and is hence state-dependent. However, as Svensson (1997) has shown a state-contingent linear inflation contract can remove the state-contingent part of the inflation bias. What we have discovered is that a nonlinear effect of the shock on the position of the trade-off along with adaptive expectations yields an opportunistic response. However, such nonlinear shift effect of $u_t$ has no theoretical underpinning implying that the theory for opportunism is weak.

**IV Conclusion**

The success of monetary policy in restoring price stability in developed economies has shifted focus on the design of monetary policy in a low inflation environment. Indeed one of the primary motivations behind an explicit inflation target was the view that by clearly communicating a low inflation objective, monetary authorities can anchor inflationary expectations, thereby reducing the cost of disinflation. Following recent arguments favouring an ‘opportunistic approach’ to disinflation, an asymmetric policy response to underlying shocks was introduced in a conventional linear-quadratic framework. It was found that such asymmetries result in higher inflation (and output) variability under commitment.

Finally, our analysis sheds light on the possible rationale as to why a policymaker would pursue such policies given that they are sub-optimal. We show that a nonlinear effect of the shock on the position of the Phillips curve trade-off along with adaptive
expectations provides justification for an opportunistic response. However, such non-linear shift effects have no theoretical underpinning thereby undermining the theory for opportunism. Thus, political economy considerations, not economic theory, stand as the strongest rationale for the opportunistic approach.
Appendix A

Nonlinear Expectation

Note that for a continuous random variable ‘x’ having a density function \( f(x) \) the expectation of ‘x’ is defined as

\[
E(x) = \int_{-\infty}^{\infty} x f(x) \, dx
\]

where the probability density function \( f(x) \) is

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}.
\]

Therefore, \( E(x) = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(\frac{\varepsilon-\mu}{\sigma})^2} \, d\varepsilon \, dx \).

Thus,

\[
E_t(e^{\alpha \varepsilon_t}) = \int_{-\infty}^{\infty} e^{\alpha \varepsilon_t} \cdot \frac{1}{\sigma_t \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(\frac{\varepsilon_t - \mu}{\sigma_t})^2} \, d\varepsilon_t
\]

\[
= \int_{-\infty}^{\infty} \frac{1}{\sigma_t \sqrt{2\pi}} \exp\left(\frac{2\sigma_t^2 \alpha \varepsilon_t - (\varepsilon_t)^2 + (E_t(\varepsilon_t))^2 - 2\varepsilon_t E_t(\varepsilon_t))}{2\sigma_t^2}\right) \, d\varepsilon_t
\]

\[
= \int_{-\infty}^{\infty} \frac{1}{\sigma_t \sqrt{2\pi}} \exp\left(\frac{(\varepsilon_t - E_t(\varepsilon_t) + \sigma_t^2 \alpha)^2 - (E_t(\varepsilon_t))^2}{2\sigma_t^2}\right) \, d\varepsilon_t
\]

\[
= \int_{-\infty}^{\infty} \frac{1}{\sigma_t \sqrt{2\pi}} \exp\left(-\frac{(\varepsilon_t - (E_t(\varepsilon_t) + \alpha \sigma_t^2))^2}{2\sigma_t^2}\right) \, d\varepsilon_t \times \exp\left(a E_t(\varepsilon_t) + \frac{a^2 \sigma_t^2}{2}\right)
\]

Note that \( E_t(\varepsilon_t) = 0 \) and

\[
\int_{-\infty}^{\infty} \frac{1}{\sigma_t \sqrt{2\pi}} \exp\left(-\frac{(\varepsilon_t - (E_t(\varepsilon_t) + \alpha \sigma_t^2))^2}{2\sigma_t^2}\right) \, d\varepsilon_t = \int_{-\infty}^{\infty} f(x) \, dx = 1.
\]

Thus, we have

\[
E_t(e^{\alpha \varepsilon_t}) = \exp\left(\frac{a^2 \sigma_t^2}{2}\right)
\]

Appendix B

Computation of Inflation Variance

\[
E(\pi_t - \pi^*)^2 = E(\pi_t - 1 \pi_t + b \varepsilon_t - \pi^*)^2
\]

\[
= E\left(-\delta \sum_{i=0}^{\infty} (1 - \phi)^i e^{\gamma \varepsilon_t - i} + \delta e^{\gamma \varepsilon_t} + b \varepsilon_t\right)^2
\]
\[
\delta^2 \sum_{i=0}^{\infty} (1 - \phi)^{2i} E(e^{2\gamma_{t-1-i}}) + \left(\frac{\delta}{\varphi}\right)^2 E(e^{2\gamma^2_{t-e}}) + b^2 E(e^2_t) \\
= -2\delta \sum_{i=0}^{\infty} (1 - \phi)^i E\left(e^{\gamma_{t-1-i} \cdot \frac{\delta^2}{\varphi} e^{-\frac{\gamma^2_{t-e}}{2}}} \right) \\
-2\delta \sum_{i=0}^{\infty} (1 - \phi)^i E\left(e^{\gamma_{t-1-i} \cdot b \varepsilon_t}\right) + 2\frac{\delta}{\varphi} E\left(e^{-\frac{\gamma^2_{t-e}}{2}} \cdot b \varepsilon_t\right)
\]

Taking expectations and using the result in appendix A yields:

\[
\delta^2 E \sum_{i=0}^{\infty} (1 - \phi)^{2i} e^{2\gamma_{t-1-i}} = \frac{\delta^2}{\varphi^2 + 2\varphi} e^{2\gamma^2_e}
\]

and 

\[
-2\delta E \sum_{i=0}^{\infty} (1 - \phi)^i e^{\gamma_{t-1-i} \cdot \frac{\delta^2}{\varphi} e^{-\frac{\gamma^2_{t-e}}{2}}} = -2 \left(\frac{\delta}{\varphi}\right)^2 e^{\gamma^2_e}.
\]

Thus we can express the variance as

\[
E(\pi_t - \pi^*)^2 = \left(\frac{\delta^2}{\varphi^2 + 2\varphi}\right) e^{2\gamma^2_e} + \left(\frac{\delta}{\varphi}\right)^2 e^{\gamma^2_e} + b^2 \sigma^2_e - 2 \left(\frac{\delta}{\varphi}\right)^2 e^{\gamma^2_e}.
\]

After simplifying this expression we have

\[
E(\pi_t - \pi^*)^2 = \delta^2 e^{\gamma^2_e} \left(\frac{\phi e^{-\frac{\gamma^2_e}{2}} - (2 - \phi)}{\phi(2 - \phi)}\right) + b^2 \sigma^2_e
\]
References


