Competition for Foreign Capital: Endogenous Objective, Public Investment and Tax

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Indira Gandhi Institute of Development Research (IGIDR)
General Arun Kumar Vaidya Marg
Goregaon (E), Mumbai- 400065, INDIA
Email (corresponding author): ajays@igidr.ac.in

Abstract

In this paper we endogenize the objective functions of the regions as well as their decision to provide public investment in a model of competition for foreign owned mobile capital. We demonstrate that the competing regions can ‘restrict race-to-the-bottom’ in tax rates by deviating away from social welfare to net tax revenue. It is optimal for a region to be fully revenue oriented even if that region’s ultimate goal is to maximize social welfare, irrespective of whether the rival region is concerned about social welfare or net tax revenue. Moreover, we demonstrate that the regions have unilateral incentive to spend on public investment, except in case of perfect spillover. In equilibrium, both the regions spend on public investment and end up with Pareto inferior outcomes.

Keywords:
Mobile Capital, Tax Competition, Public Investment, Revenue Orientation, Social Welfare.

JEL Code:
F21, H25, R50, H40, D60

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Rupayan Pal† and Ajay Sharma‡

†, ‡ Indira Gandhi Institute of Development Research (IGIDR), India

Abstract

In this paper we endogenize the objective functions of the regions as well as their decision to provide public investment in a model of competition for foreign owned mobile capital. We demonstrate that the competing regions can restrict ‘race-to-the-bottom’ in tax rates by deviating away from social welfare to net tax revenue. It is optimal for a region to be fully revenue oriented even if that region’s ultimate goal is to maximize social welfare, irrespective of whether the rival region is concerned about social welfare or net tax revenue. Moreover, we demonstrate that the regions have unilateral incentive to spend on public investment, except in case of perfect spillover. In equilibrium, both the regions spend on public investment and end up with Pareto inferior outcomes. Key words: Mobile Capital, Tax Competition, Public Investment, Revenue Orientation, Social Welfare. JEL Classifications: F21, H25, R50, H40, D60

1 Introduction

Existing models of interregional competition for mobile capital either assume that the governments’ strategies are based on the principle of social welfare maximization or the governments are assumed to be concerned only about tax revenue collected. However, the

1Corresponding Address: Ajay Sharma, Indira Gandhi Institute of Development Research (IGIDR), Film City Road, Gen A. K. Vaidya Marg, Goregaon (East), Mumbai 400065, India. E-mails: † rupayan@igidr.ac.in, rupayampal@gmail.com; ‡ ajays@igidr.ac.in., Telephone: +91-22-28416545, Fax: +91-22-28402752.
choice of government’s objective function, social welfare or tax revenue, is likely to affect equilibrium outcomes. It is often argued that the political structure of a country plays significant role in determining its government’s objective function. Edwards and Keen (1996) and Wilson (2005), for example, argue that a leviathan government tends to maximize its net tax revenue to increase in government size so that more revenue is at the disposal of the government. Following this view, a number of authors have considered that governments maximize revenue in their models of tax competition (see, for example, Kanbur and Keen (1993), Janeba (2000), Dembour and Wauthy (2009), Marceau et al. (2010), to name a few). On the other hand, Hoyt (1991), Hindriks et al. (2008), and Matsumoto (2010), among others, subscribe to the view that governments are benevolent and maximize social welfare. In both of these two sets of papers, it is assumed that the government’s objective function, based on which optimal strategies are determined, is exogenously given. Another strand of literature incorporates political competition into the models of tax competition, where the utility function of the elected policy maker (representative citizen) serves as the objective function to determine the optimal strategies to compete for mobile capital (Persson and Tabellini (1992); Fuest and Huber (2001); Perroni and Scharf (2001); Ihori and Yang (2009). However, none of these studies recognizes the possible implications of strategic interaction among governments on the choice of their objective functions. As in case of strategic managerial delegation (Vickers (1985); Fershtman and Judd (1987); Sklivas (1987), it may be optimal for the governments to deviate from their ultimate goals and determine the competition strategies based on strategically chosen objective functions. In other words, strategic interaction among governments may induce them to deviate from their ultimate goals while deciding the optimal strategies to attract mobile capital. To the best of our knowledge, the issue of strategic determination of governments’ objective functions has not received much attention in the literature so far. This paper attempts to fill this gap in the literature.

Moreover, most of the studies consider that regions compete for mobile capital only in terms of tax rates, although it is well documented that productivity enhancing public investment in a region enhances its prospect to attract capital. Recently, few studies,
such as Hindriks et al. (2008), Zissimos and Wooders (2008), Dembour and Wauthy (2009), Kotsogiannis and Serfes (2010) and Pieretti and Zanaj (2011), have enlarged the strategy space of the competing regions in order to examine the implications of competition in terms of both tax rates and public investment. However, in these studies the decision of whether to spend on public investment or not is treated as exogenous. The question is, is it always optimal for a region to spend on public investment?

This paper offers a model of intraregional competition for foreign owned mobile capital, where the objective functions of the governments as well as the decision to spend on productivity enhancing public investment are endogenously determined. We consider that there are two regions competing for foreign owned mobile capital. Each region strategically decides its capital tax rate. In addition, regions may decide to spend on productivity enhancing public investment, which has spillover effect. It is evident that higher tax rate in a region makes it less attractive destination for mobile capital compared to its rival region. However, that region may decide to spend on public investment and makes it more appealing destination for capital in spite of its higher tax rate, unless there is perfect spillover of public investment. We show that it is optimal for each region to be fully revenue oriented, even if its ultimate goal is to maximize social welfare, irrespective of whether the rival region is concerned about social welfare or net tax revenue. This result holds true irrespective of whether the regions decide to spend on public investment or not. In other words, it is always optimal for the competing regions to choose their respective net tax revenue maximizing strategies. The intuition behind this result is as follows. Increase in social welfare orientation of a region makes it more aggressive in tax competition, which in turn induces it to reduce its tax rate more than proportionately than the reduction in its rival’s tax rate. As a result, loss of net tax revenue of the more social welfare oriented region, due to reduction in its tax rate, is greater than its gains from returns to immobile factors, due to increased capital flow in that region. When the regions spend on public investment, higher social welfare orientation of a region induces it (its rival) to spend more (less) on public investment as well as to set lower tax rate than its rival, in spite of the positive effect of public investment of a region on its tax rate. Therefore, both net tax
revenue as well as social welfare of a region are decreasing in its social welfare orientation. These are new insights. This paper also demonstrates that the competing regions can restrict race-to-the-bottom in tax rates by deviating away from social welfare to net tax revenue.

Moreover, we show that the regions have unilateral incentive to spend on public investment, unless the spill over is perfect. The reason is, if a region provides public investment, the other region needs to counteract that by reducing tax and spending on public investment, since only tax reduction is sub-optimum from both social welfare and net tax revenue point of view. On the other hand, if a region does not spend on public investment, by providing some level of public investment the other region can increase tax rate to some extent and still attract more mobile capital, which in turn lead to higher net tax revenue as well as higher returns to immobile factor of the other region. However, when both the regions spend on public investment, positive effect of a region’s public investment on its attractiveness cancels out due to the negative effect of its rival’s spending on public investment. Therefore, regions face a Prisoners’ dilemma type of situation while taking decision about public investment. In equilibrium, both the regions spend on public investment and end up with Pareto inferior outcomes.

The rest of this paper is organized as follows. The next section presents the basic model. Section 3 develops the equilibrium analysis in absence of public investment. Section 4 proceeds with the endogenous determination of public investment and reexamines the issue of strategic determination of governments’ objective functions. Section 5 concludes.

2 The Model

Suppose that there are two regions, region-1 and region-2, competing for foreign owned mobile investment capital of total amount one, which is exogenously determined, in order to maximize their respective objectives. Each region decides the tax rate $t_i \geq 0$ on mobile capital $x_i$ ($0 \leq x_i \leq 1$) and the level of public investment $g_i \geq 0$, $i = 1, 2$. Higher tax rate on capital in any region dampens the flow of capital in that region, but that may lead
to higher tax revenue. In contrast, public investment in any region facilitates production in both the regions and, thus, it enhances productivity of capital across regions. However, the effect of public investment \((g_i)\) in region \(i\) on productivity of capital in \(i\)th region is higher than that in the \(j\)th region, unless there is perfect spillover of public investment. The cost to provide public investment \(g_i\) by region \(i\) is assumed to be \(\frac{g_i^2}{2}\), \(i = 1, 2\). So, the net tax revenue of region \(i\) is as follows.

\[
NT_i = t_i x_i - \frac{g_i^2}{2}, \quad i = 1, 2.
\] (1)

Following Hindriks et al. (2008), we consider that the production function of a region \(i\) \((= 1, 2)\) is as follows.

\[
F_i(x_i, g_i) = \left(\gamma + g_i + \theta g_j\right)x_i - \frac{\delta x_i^2}{2}, \quad i, j = 1, 2, \quad i \neq j,
\] (2)

where \(x_i\) is the amount of mobile capital invested in region \(i\),\(^2\) \(\gamma \ (> 0)\) is the technology parameter, \(\delta \ (> 0)\) denotes the rate of decline in the marginal productivity of mobile capital, and \(\theta \ (0 \leq \theta \leq 1)\) is the spillover effect of public investment in one region to the other region’s productivity. Higher value of \(\theta\) denotes higher spillover effect; \(\theta = 1\, (\theta = 0)\) corresponds to the extreme case of perfect (no) spillover. Clearly, regions have symmetric production functions, which are increasing, twice continuously differentiable and concave in the level of capital. We assume that \(\gamma > \delta > 1\). The first part of the inequality, i.e., \(\gamma > \delta\), ensures that marginal productivity of capital is always positive.\(^3\) The second part of the inequality, i.e, \(\delta > 1\), ensures existence and stability of interior solutions in all the cases considered.

Assuming that the capital market is perfectly competitive\(^4\) and normalizing the price of output to be one, we can write the returns to immobile factors of region \(i\) as, \(IR_i = [F_i(.) - x_i \frac{\partial F_i(.)}{\partial x_i}] = \frac{\delta x_i^2}{2}\). Clearly, returns to immobile factors in a region is increasing in

\(^2\)\(x_i\) can also be interpreted as the share of mobile capital invested in region \(i\).

\(^3\)Note that, in absence of any tax and public investment, if full amount of mobile capital is invested in any one of the two regions, marginal productivity of capital in that region is equal to \(\gamma - \delta\).

\(^4\)It implies that capital is paid according to its marginal productivity.
investment of mobile capital in that region, at an increasing rate $\delta$. Since mobile investment capital is foreign owned, social welfare ($SW$) of a region is given by the sum of returns to immobile factors ($IR$) and net tax revenue ($NT$) of that region:

$$SW_i = IR_i + NT_i = \frac{\delta}{2}x_i^2 + \left[t_ix_i - \frac{g^2_i}{2}\right], \quad i = 1, 2. \quad (3)$$

Note that the parameter $\delta$ can also be interpreted as the rate of increase in ‘marginal social welfare’ ($\frac{\partial SW_i}{\partial x_i}$) of a region due to increase in mobile capital in that region. The above formulation of social welfare function is in line with Kempf and Rota-Graziosi (2010), Hindriks et al. (2008) and Laussel and Le Breton (1998).\(^5\)

We consider that a region may be either interested only in net tax revenue ($NT$) or it may be interested in social welfare ($SW$). We refer to these two types of regions as ‘fully revenue oriented region’ and ‘fully social welfare oriented’, respectively. To illustrate it further note that if a region is debt constraint, its primary concern may be to generate as much net tax revenue as possible. Otherwise, the region may be concerned about the returns to immobile factors as well as net tax revenue.\(^6\) We allow for the possibility that regions can have different ultimate goals.

It is evident that, in absence of competition for mobile capital, i.e., if there is only one region, it is optimal for a region to decide the tax rate and level of public investment that maximizes its $NT$ or $SW$, depending on whether the region is fully revenue oriented or fully social welfare oriented. That is, in absence of any competition, a fully revenue (social welfare) oriented region would try to maximize $NT$ ($SW$) directly. Any deviation from that would result in suboptimal solution. The question is, will it remain valid in case of more than one region? When regions compete for mobile capital, effects of strategic interaction between regions may render deviation from ultimate goals to be beneficial. Then, how would the regions decide the tax rate (or tax rate and public investment) when there is competition for mobile capital? We assume that, in order to achieve maximum net

\(^5\)For further justifications of the objective functions of regions see Laussel and Le Breton (1998).
\(^6\)We note here that, other than fiscal position, institutional and political factors may also play crucial roles to determine a region’s ultimate goal.
tax revenue or maximum social welfare, region \( i (= 1, 2) \) considers the following objective function in order to determine its optimum strategies, ‘\( t_i \)’ or ‘\( g_i \)’.

\[
O_i = \alpha_i SW_i + (1 - \alpha_i) NT_i; \quad 0 \leq \alpha_i \leq 1, \ i = 1, 2,
\]

\[
= \alpha_i IR_i + NT_i, \quad \text{since} \ SW_i = IR_i + NT_i
\]

where \( \alpha_i \)s are decided by the regions simultaneously and independently. Note that, if \( \alpha_i = 1 \) (\( \alpha_i = 0 \)), \( O_i = SW_i \) (\( O_i = NT_i \)). That is, whether a region would deviate from its ultimate goal or not, while deciding its optimum strategies, that depends on the equilibrium value of \( \alpha_i \).

The above formulation of regions’ objective functions can also be viewed as a case of strategic delegation of authority. We may consider that the central authority of a region delegates the task to decide the tax rate and level of public investment to a risk-neutral manager, who may be a bureaucrat or a minister, and offers the incentive structure as in \( (4) \) to the manager. Then, given the incentive structure, the manager will maximize \( O_i \).

By choosing the incentive parameter \( \alpha_i \) appropriately, the region \( i \) can induce its manager to be more or less aggressive competitor. If the equilibrium value of \( \alpha_i \) is such that \( O_i \) coincides with region \( i \)’s ultimate goal, delegation is not desirable in region \( i \). Otherwise, region \( i \) would benefit through delegation.

The stages of the game involved are as follows.

**Stage 1:** Region 1 and region 2 simultaneously and independently decide their respective objective functions by choosing the values of \( \alpha_1 \) and \( \alpha_2 \), respectively, given their ultimate goals.

**Stage 2:** Both the regions simultaneously and independently decide whether to spend on public investment or not.

**Stage 3:** Regions are engaged in simultaneous move tax competition or in simultaneous move competition in terms of both tax rate and public investment.

**Stage 4:** Owners of mobile capital decide how much to invest in which region.
We start from the fourth stage by noting that the allocation of capital between the two regions depends on productivity of capital and tax rate of each region. Since, capital market is perfectly competitive, marginal return to capital net of tax in region \( i \) is 
\[
F'_{i,x_i}(x_i, g_i) - t_i, \quad i = 1, 2, \text{ if region } i \text{ gets } x_i \text{ amount of mobile capital.}
\]
It implies that we must have 
\[
F'_{i,x_i}(x_i, g_i) - t_i > 0, \quad \text{for region } i \text{ to get } x_i \text{ amount of mobile capital, considering region } i \text{ in isolation.}
\]
Note that, for any given allocation of capital, if net marginal returns to capital differ between regions, reallocation of capital from the region with lower net return to the other region takes place. Therefore, to rule out the possibility of arbitrage, we need to have the allocation such that net marginal return to capital is same in both the regions. For feasibility of such arbitrage-proof allocation of capital, \( x_1 + x_2 \leq 1 \) must be satisfied. We consider that entire amount of mobile capital is allocated between region 1 and region 2: \( x_1 + x_2 = 1 \). In other words, we rule out the possibility of mobile capital to remain idle. Therefore, the arbitrage-proof equilibrium allocation of mobile capital, for any given levels of public investments and tax rates, between the two regions is given by

\[
F'_{1,x_1}(x_1, g_1) - t_1 = F'_{2,x_2}(x_2, g_2) - t_2 > 0, \quad \text{(5)}
\]
and \( x_1 + x_2 = 1. \) \( \quad \text{(6)} \)

From (5) and (6), we get the equilibrium investment of mobile capital in region 1 and region 2, given the levels of public investments and tax rates, as follows.

\[
x_1 = \frac{1}{2} + \frac{1}{2\delta} [(t_2 - t_1) + (1 - \theta)(g_1 - g_2)] \quad \text{(7a)}
\]
\[
x_2 = \frac{1}{2} - \frac{1}{2\delta} [(t_2 - t_1) + (1 - \theta)(g_1 - g_2)] \quad \text{(7b)}
\]

Clearly, the allocation of mobile capital depends on each region’s tax rate as well as public investment. Increase in tax rate of one region negatively (positively) affects the flow of mobile capital in that (the other) region: \( \frac{\partial x_i}{\partial t_i} < 0 \) and \( \frac{\partial x_i}{\partial t_i} > 0; \quad i, j = 1, 2, \ i \neq j. \) In contrast, increase in public investment in one region increases (decreases) capital flow in that (the other) region, unless there is perfect spillover of public investment: \( \frac{\partial x_i}{\partial g_i} > 0 \) and \( \frac{\partial x_i}{\partial g_i} < 0, \) unless \( \theta = 1; \ i, j = 1, 2, \ i \neq j. \)
From (7a), (7b) and (4), we get $O_1 = O_1(t_1, t_2, g_1, g_2, \alpha_1)$ and $O_2 = O_2(t_1, t_2, g_1, g_2, \alpha_2)$. It is easy to check that \[ \frac{\partial^2 O_1}{\partial t_1 \partial t_2} = \frac{2 - \alpha_1}{4\delta} > 0 \] and \[ \frac{\partial^2 O_2}{\partial t_2 \partial t_1} = \frac{2 - \alpha_2}{4\delta} > 0, \forall \alpha_1, \alpha_2 \in [0, 1]. \] It implies that the marginal effect of one region’s tax rate on its own payoff increases with the increase in other region’s tax rate. Therefore, tax rates, $t_1, t_2$, are strategic complements. On the other hand, we can also check that \[ \frac{\partial^2 O_1}{\partial g_1 \partial g_2} = \frac{-\alpha_1(1-\theta)^2}{4\delta} < 0 \] and \[ \frac{\partial^2 O_2}{\partial g_2 \partial g_1} = \frac{-\alpha_2(1-\theta)^2}{4\delta} < 0, \forall \alpha_1, \alpha_2 \in (0, 1] \text{ and } \theta \in [0, 1). \] That is, the marginal effect of public investment in one region on that region’s payoff is decreasing in other region’s public investment, unless there is perfect spillover of public investment or regions directly maximizes net tax revenue. So, in contrast to the tax rates, public investments ($g_1, g_2$) are strategic substitutes.

**Lemma 1:** When regions compete for foreign owned mobile capital in terms of tax rate and productivity enhancing public investment, the two strategies are of opposite nature. Tax rates, $t_1$ and $t_2$, are strategic complements. Whereas, levels of public investments, $g_1$ and $g_2$, are strategic substitutes, unless there is perfect spillover and regions are net tax revenue oriented.

It seems to be interesting to examine the implications of possible interplays between the two strategies of opposite nature on equilibrium outcomes. From Lemma 1, it is evident that regions’ tax-reaction curves, in $t_1 - t_2$ plane, would be upward slopping. However, the reaction curves for public investments, in $g_1 - g_2$ plane, would be downward slopping. It seems to indicate that in response to under cutting tax rate by region 1 (say), region 2 can either under cut its tax rate and/or increase its level of public investment in order to maintain the status quo or to attract mobile capital in region 2. Therefore, it is not necessary that there would be race-to-the-bottom in tax rates. We explore this issue further in subsequent sections.
3 Regions do not spend on public investment

To begin with, we consider that none of the regions spends in public investment: \( g_1 = g_2 = 0 \). In other words, we assume that the two regions compete only in terms of tax rates. Therefore, the stage 4 equilibrium outcomes are given by, \( x_1 = \frac{1}{2} + \frac{1}{2\delta} [t_2 - t_1] \) and \( x_2 = \frac{1}{2} - \frac{1}{2\delta} [t_2 - t_1] \). Plugging these expressions in (4), we obtain \( O_i = \frac{(\delta - t_i + t_j)(4 - \alpha_i) + \alpha_i (\delta + t_j)}{8\delta} \); \( i, j = 1, 2 \) and \( i \neq j \). Now, note that, given any \((\alpha_1, \alpha_2)\), the outcome of strategic interaction between the two regions in stage 3 is given by the following two equations:

\[
\begin{align*}
\frac{\partial O_1}{\partial t_1} = 0 & \Rightarrow t_1 = \frac{(2 - \alpha_1)(\delta + t_2)}{4 - \alpha_1} \quad (8a) \\
\text{and} \quad \frac{\partial O_2}{\partial t_2} = 0 & \Rightarrow t_2 = \frac{(2 - \alpha_2)(\delta + t_1)}{4 - \alpha_2} \quad (8b)
\end{align*}
\]

In the above, equations (8a) and (8b) represent the tax reaction functions (TRFs) of region 1 and region 2, respectively. Clearly, the TRFs of the regions are upward sloping in \((t_1, t_2)\) plane. Figure 1 depicts the tax reaction functions of the two regions. We can solve (8a) and (8b) for the stage 3 equilibrium tax rates as

\[
\begin{align*}
t_1 & = \frac{(2 - \alpha_1)(3 - \alpha_2)\delta}{6 - \alpha_1 - \alpha_2} \quad (9a) \\
\text{and} \quad t_2 & = \frac{(2 - \alpha_2)(3 - \alpha_1)\delta}{6 - \alpha_1 - \alpha_2} \quad (9b)
\end{align*}
\]

Clearly, an increase in the extent of social welfare orientation of any region, i.e., an increase in \( \alpha_i \), reduces the tax rates of both the regions: \( \frac{\partial t_i}{\partial \alpha_i} = -\frac{(3-\alpha_i)(\delta + t_2)}{(6-\alpha_1-\alpha_2)^2} < 0 \) and \( \frac{\partial t_j}{\partial \alpha_i} = -\frac{(2-\alpha_j)(3-\alpha_i)\delta}{(6-\alpha_1-\alpha_j)^2} < 0, \ i \neq j \). It is easy to check that the impact of a change in social welfare orientation of region \( i \) on region \( i \)'s tax rate (own-effect) is higher than that on region \( j \)'s tax rate (cross-effect): \( |\frac{\partial t_i}{\partial \alpha_i}| > |\frac{\partial t_j}{\partial \alpha_i}| \). That is, an increase in social welfare orientation of a region induces that region to reduce its tax rate more than proportionately to the reduction of its rival’s tax rate. Therefore, higher social welfare orientation of any region

\[\text{Note that second order conditions for maximization are satisfied: } \frac{\partial^2 O_i}{\partial t_i^2} = -\frac{4-\alpha_1}{4\delta} < 0, \ \frac{\partial^2 O_2}{\partial t_2^2} = -\frac{4-\alpha_2}{4\delta} < 0 \text{ and } \frac{\partial^2 O_1}{\partial t_2^2} - \frac{\partial^2 O_1}{\partial t_1 \partial t_2} = \frac{6-\alpha_1-\alpha_2}{4\delta^2} > 0, \text{ since } 0 \leq \alpha_1, \alpha_2 \leq 1.\]
leads to more intense tax competition, which in turn leads to lower tax rates in equilibrium. In other words, race-to-the-bottom in case of pure tax competition is intensified in case the regions are more social welfare oriented. It also implies that, regions can potentially restrict the race-to-the-bottom by moving away from social welfare maximization towards revenue maximization. One way to do that is to delegate the task to decide the tax rate to an authority by offering a revenue oriented incentive scheme. Alternatively, a region can perceive that a weighted average of net tax revenue and social welfare, with higher weight given to net tax revenue, as its objective function while competing for mobile capital.

To illustrate this further, note that, due to a decrease in $\alpha_i$, the tax reaction function of region $i$ rotates towards $t_i$-axis and shifts outwards, see Figure 1. That is, for any given $t_j$, region $i$ chooses higher tax rate ($t_i$), if it is more revenue oriented. In case of delegation, by putting higher weight to net tax revenue while designing the incentive scheme for the delegated authority, a region can make its delegated authority to be less aggressive in tax competition. Then the question is, will it be optimal for a region to deviate away from
social welfare towards net tax revenue? Does that depend on the ultimate goal (NT or SW) of the region? How does the equilibrium look like, if the two regions are asymmetric in terms of their ultimate goals?

Note that the net tax revenue (NT) of a region is decreasing in its extent of social welfare orientation.

\[
\frac{\partial (NT_i)}{\partial \alpha_i} = \frac{\partial (t_i x_i)}{\partial \alpha_i} = \frac{\partial (t_i x_i)}{\partial t_i} \frac{\partial t_i}{\partial \alpha_i} + \frac{\partial (t_i x_i)}{\partial t_j} \frac{\partial t_j}{\partial \alpha_i} = \frac{\alpha_i (3 - \alpha_j)}{2 (6 - \alpha_i - \alpha_j)} \frac{\partial t_i}{\partial \alpha_i} + \frac{(2 - \alpha_i) (6 - \alpha_i) (3 - \alpha_j)}{2 (4 - \alpha_i) (6 - \alpha_i - \alpha_j)} \frac{\partial t_j}{\partial \alpha_i} = \frac{-(3 - \alpha_j)^2 (2 + \alpha_i - \alpha_j) \delta}{(6 - \alpha_i - \alpha_j)^3} < 0
\]

It implies that, if region \(i\)'s ultimate goal is to maximize net tax revenue, it has unilateral incentive to reduce \(\alpha_i\) to zero and be completely revenue oriented. Therefore, if the regions' ultimate goals are to maximize respective net tax revenues, it is optimal for both the regions to set \(\alpha = 0\) in stage 1. That is, in this case, it is optimal for both the regions to choose their respective net tax revenue maximizing tax rates. We summarize the equilibrium outcomes, corresponding to the present scenario, in Lemma 2.

**Lemma 2:** When the two regions compete for foreign owned mobile capital only in terms of tax rates and the regions’ ultimate goals are to maximize their respective net tax revenues, the equilibrium incentive parameters, tax rates, capital allocation, net tax revenues and social welfare are, respectively, \(\alpha^*_i = \alpha^*_2 = 0\), \(t^*_1 = t^*_2 = \delta\), \(x^*_1 = x^*_2 = \frac{1}{2}\), \(NT^*_1 = NT^*_2 = \frac{\delta}{2}\) and \(SW^*_1 = SW^*_2 = \frac{5\delta}{8}\).

Note that, if regions perceive their respective social welfare as their objective functions while deciding the tax rates, i.e., if \(\alpha_1 = \alpha_2 = 1\), the tax rates, capital allocation, net tax revenues and social welfare are as follows: \(t_1 = t_2 = \frac{\delta}{2}\), \(x_1 = x_2 = \frac{1}{2}\), \(NT_1 = NT_2 = \frac{\delta}{4}\) and \(SW_1 = SW_2 = \frac{3\delta}{8}\). Clearly, tax rate, net tax revenue and social welfare of each region is lower in this case compared to that in Lemma 2.
Now, let’s turn to the scenario where the two regions’ ultimate goals are to maximize their respective SWs. In this case, the problem of region $i$ in stage 1 can be written as $\max_{\alpha_i} \text{SW}_i(\alpha_i, \alpha_j) = IR_i(\alpha_i, \alpha_j) + NT_i(\alpha_i, \alpha_j)$. Now, note that

$$\frac{\partial IR_i}{\partial \alpha_i} = \delta x_i \frac{\partial x_i}{\partial \alpha_i}$$

$$= \delta x_i \left[ \frac{\partial x_i}{\partial \alpha_i} \frac{\partial t_i}{\partial \alpha_i} + \frac{\partial x_i}{\partial \alpha_i} \frac{\partial t_j}{\partial \alpha_i} \right]$$

$$= \delta x_i \left[ \frac{1}{2\delta} \left( \frac{\partial t_j}{\partial \alpha_i} - \frac{\partial t_i}{\partial \alpha_i} \right) \right]$$

$$= \frac{(3 - \alpha_j) \delta}{(6 - \alpha_i - \alpha_j)^2}$$

$$= \frac{(3 - \alpha_j)^2 \delta}{(6 - \alpha_i - \alpha_j)^3} > 0.$$

So, returns to immobile factors in region $i$ ($IR_i$) increases with the increase in region $i$’s orientation towards social welfare. On the other hand, as noted before, tax revenue of a region is decreasing in that region’s social welfare orientation ($\frac{\partial NT_i}{\partial \alpha_i} < 0$). Therefore, the net effect of a region’s social welfare orientation on its social welfare depends on the relative magnitudes of these two opposing effects. It turns out that the effect of increase in social welfare orientation of a region on its net tax revenue dominates that on returns to immobile factors: $|\frac{\partial NT_i}{\partial \alpha_i}| > |\frac{\partial IR_i}{\partial \alpha_i}|$. As a result, $\frac{\partial \text{SW}_i}{\partial \alpha_i} = -\frac{(3 - \alpha_j)^2(1 + \alpha_i - \alpha_j) \delta}{(6 - \alpha_i - \alpha_j)^3} < 0$. Thus, $\max_{\alpha_i} \text{SW}_i(\alpha_i, \alpha_j) \Rightarrow \alpha_i = 0$. That is, a region has unilateral incentive to be revenue oriented, in spite of the fact that its ultimate goal is to maximize social welfare. In equilibrium, both the regions set their net tax revenue maximizing tax rates, which in turn maximizes their respective social welfare. In other words, in case of competition for mobile capital in terms of tax rates, net tax revenue maximizing tax rates maximizes social welfare. The reason is, regions can restrict wasteful race-to-the-bottom in tax rates by being revenue oriented, i.e, by choosing $\alpha_1 = \alpha_2 = 0$. Clearly, the equilibrium outcomes in this case also would be same as reported in Lemma 2.

**Proposition 1:** Net tax revenue maximizing tax rate maximizes social welfare, if the two regions compete for foreign owned mobile capital only in terms of tax rates and both
the regions ultimate goal is to maximize social welfare.

From Proposition 1 and Lemma 2 we can say that, in case of symmetric ultimate goals of the regions, it is optimal for each region to consider its net tax revenue as its objective function while competing in terms of tax rate. It implies that, no delegation is optimal, if the ultimate goal of a region is to maximize its net tax revenue. This is same as in case of no strategic interaction between regions. In contrast, delegation through revenue oriented incentive scheme is optimal in case of tax competition, if social welfare maximization is the ultimate goal. In other words, it is optimal for both the regions to perceive respective NTs as their objective functions, although their ultimate goals are to maximize respective SWs.

To illustrate it further, note that, if the regions’ ultimate goals are the same, in equilibrium, they would perceive the same objective function ($\alpha_1 = \alpha_2$) and set the same tax rate ($t_1 = t_2$). As a result, mobile capital would be equally divided between the two regions ($x_1 = x_2 = \frac{1}{2}$) in equilibrium. It implies that race-to-the-bottom in terms of tax rates reduces tax revenue of each region, keeping the returns to immobile factors (IR) unchanged. Therefore, irrespective of whether the regions’ ultimate goal is to maximize their respective SWs or NTs, it is optimal for the symmetric regions to be fully revenue oriented.

Finally, lets turn to the case of asymmetric regions, i.e, when the two regions have different ultimate goals. Without any loss of generality, we consider that the ultimate goal of region $i$ is to maximize its social welfare, whereas region $j$ ($\neq i$) wants to maximize its net tax revenue. From the above discussion, it is evident that $\frac{\partial SW_i}{\partial \alpha_i} < 0$ and $\frac{\partial NT_j}{\partial \alpha_j} < 0$, $\forall i, j = 1, 2$, irrespective of whether the regions are symmetric or not. That is, each region has unilateral incentive to be revenue oriented, irrespective of its ultimate goal and whether its rival has different ultimate goal or not. Therefore, in case of asymmetric regions also, in equilibrium, each region perceives its net tax revenue as its objective function while deciding the tax rate. In other words, it is optimal only for region $i$, not for region $j$, to delegate the task to decide tax rate to an authority, and region $i$ offers a fully net tax revenue oriented incentive scheme to the delegated authority. Clearly, in this case also, the
equilibrium outcomes are same as that in Lemma 2.

**Proposition 2:** In equilibrium, each region sets its net tax revenue maximizing tax rate, irrespective of whether its ultimate goal is to maximize net tax revenue or social welfare and whether the ultimate goal of its rival is different from its own or not, when the two regions are engaged in tax competition.

### 4 Endogenous determination of public investment

Unlike as in section 3, we now allow the levels of public investments to be endogenously determined. Note that higher level of public investment in a region attracts mobile capital in that region. Therefore, sufficiently higher level of public investment in a region may nullify negative impact of higher tax rate in that region on allocation of mobile capital. Also, note that the levels of public investments in the two regions are strategic substitutes, while tax rates are strategic complements. Moreover, to provide public investment regions need to incur some cost, and that cost is increasing in level of public investment. Therefore, the questions are as follows. Is it optimal for a region to spend on public investment? How does the equilibrium look like when the two regions compete for mobile capital in terms of both tax rate and public investment? Does it remain optimal for a region, whose ultimate goal is to maximize its social welfare, to be revenue oriented in case of multidimensional competition for mobile capital?

Now, in this case the allocation of mobile capital between the two regions, in stage 4, are given by (7a) and (7b). Therefore, given the extents of social welfare orientation of the two regions, $(\alpha_1, \alpha_2)$, the problem of region $i$ in stage 3 can be written as follows.

\[
Max_{t_i, g_i} O_i = \alpha_i \frac{\delta}{2} x_i^2 + [t_i x_i - g_i^2] \\
\text{subject to} \\
x_i = \frac{1}{2} + \frac{1}{2\delta} [(t_j - t_i) + (1 - \theta)(g_i - g_j)]; i \neq j
\]
Therefore, the outcomes of strategic interactions between the two regions in stage 3 are given by the following equations.\(^8\)

\[
\frac{\partial O_1}{\partial t_1} = 0 \Rightarrow t_1 = \frac{(2 - \alpha_1) \left[ \delta + (1 - \theta) \left( g_1 - g_2 \right) + t_2 \right]}{4 - \alpha_1} \quad (10a)
\]

\[
\frac{\partial O_1}{\partial g_1} = 0 \Rightarrow g_1 = \frac{(1 - \theta) \left[ \alpha_1 \delta + (2 - \alpha_1) t_1 + \alpha_1 t_2 - \alpha_1 (1 - \theta) g_2 \right]}{4 \delta - \alpha_1 (1 - \theta)^2} \quad (10b)
\]

\[
\frac{\partial O_2}{\partial t_2} = 0 \Rightarrow t_2 = \frac{(2 - \alpha_2) \left[ \delta + (1 - \theta) \left( g_2 - g_1 \right) + t_1 \right]}{4 - \alpha_2} \quad (11a)
\]

\[
\frac{\partial O_2}{\partial g_2} = 0 \Rightarrow g_2 = \frac{(1 - \theta) \left[ \alpha_2 \delta + (2 - \alpha_2) t_2 + \alpha_2 t_1 - \alpha_2 (1 - \theta) g_1 \right]}{4 \delta - \alpha_2 (1 - \theta)^2} \quad (11b)
\]

For exogenously given \(g_1\) and \(g_2\), the tax reaction functions of region 1 and region 2 are given by (10a) and (11a), respectively. Clearly, for any given \(g_2\), if \(g_1\) increases, the tax reaction function of region 1 (TRF\(_1\)), in \(t_1 - t_2\) plane, shifts out and the tax reaction function of region 2 (TRF\(_2\)) shifts down, as depicted in Figure 2. As a result, the equilibrium tax rate of region 1 increases, while the tax rate of region 2 decreases. To be explicit, note that, for any given \(g_1\) and \(g_2\), the equilibrium tax rates are as follows: \(t_1(g_1, g_2) = \frac{(2 - \alpha_1)(3 - \alpha_2) \delta + (1 - \theta)(g_1 - g_2)}{6 - \alpha_1 - \alpha_2}\) and \(t_2(g_1, g_2) = \frac{(2 - \alpha_2)(3 - \alpha_1) \delta - (1 - \theta)(g_1 - g_2)}{6 - \alpha_1 - \alpha_2}\). Clearly, \(\frac{\partial t_1(g_1, g_2)}{\partial g_1} > 0\), but \(\frac{\partial t_2(g_1, g_2)}{\partial g_1} < 0\). The reason is, if there is an increase in \(g_1\) and tax rates are same across regions, region 1 becomes more attractive destination of mobile capital. Such comparative advantage of region 1 enables it to charge higher tax rate. On the other hand, comparative disadvantage of region 2, due to increase in \(g_1\), induces it to decrease its tax rate.

On the other hand, given any tax rates \(t_1\) and \(t_2\), the public investment reaction functions of region 1 and region 2 are given by (10b) and (11b), respectively. It is easy to check that public investment reaction functions of the regions are downward sloping. This is because public investments are strategic substitutes, as discussed in Section 2. We depict the public investment reaction function of region 1, denoted by IRF\(_1\), and that of region 2, denoted by IRF\(_2\), in Figure 3. Corresponding to some particular tax rates, the equi-

\(^8\)The second order condition for maximization and the stability condition are satisfied, since \(\delta > 1\) by assumption.
Figure 2: Change in public investment and tax reaction functions

librium pair of public investments is denoted by point E in Figure 3. Now, note that, if
tax rate of any one region increases, the public investment reaction function of both the
regions shifts out and the equilibrium changes from point E to E'. Interestingly, if there
is an increase in tax rate of region i, equilibrium public investment of both the regions
increases. Solving the (10b) and (11b), we get the equilibrium public investments, given
the tax rates, as follows.

\[ g_1 = \frac{(1-\theta)[\alpha_1(2\delta-\alpha_2(1-\theta)^2) + t_1 + \alpha_1(2\delta-(1-\theta)^2)t_2]}{2\delta(4\delta-(\alpha_1+\alpha_2)(1-\theta)^2)} \]

\[ g_2 = \frac{(1-\theta)[\alpha_2(2\delta-\alpha_1(1-\theta)^2) + t_2 + \alpha_2(2\delta-(1-\theta)^2)t_1]}{2\delta(4\delta-(\alpha_1+\alpha_2)(1-\theta)^2)} \]

It is easy to check that both g₁ and g₂ are increasing in t₁: \( \frac{\partial g_1}{\partial t_1} > 0 \) and \( \frac{\partial g_2}{\partial t_1} > 0 \), since \( \delta > 1 \). The intuition is as follows.
If there is an increase in region 1’s tax rate, region 1 becomes relatively less attractive
destination of mobile capital. As a result, the effect of increase in public investment in
region 2 on its welfare is now higher than that in case there was no increase in region 1’s
tax rate. Thus, for any given t₂ and g₁, if t₁ increases, region 2 spends more on public
investment. On the other hand, increase in t₁ induces region 1 to spend more in public
investment. Therefore, equilibrium public investment of both the regions increases with
increase in tax rate of any region.

Now, we turn to examine whether regions have unilateral incentive to spend on public
investment or not. It is easy to check that, for any given level of public investment in region
Figure 3: Change in tax rate and public investment reaction functions

$j$, allocation of capital in region $i$ is increasing in public investment of region $i$, $\frac{\partial x_i}{\partial g_i} > 0$ for any $g_j$, provided that $0 \leq \theta < 1$; $i, j = 1, 2, i \neq j$.\footnote{Since, $x_i = \frac{1}{2} + \frac{1}{2\theta}(t_j - t_i) + (1 - \theta)(g_i - g_j)$ and $t_i(g_i, g_j) = \frac{(2 - \alpha_i)(3 - \alpha_j) + (1 - \theta)(g_i - g_j)}{6 - \alpha_i - \alpha_j}$ ($i, j = 1, 2; i \neq j$),}

Therefore, for any given $g_j$, we have

$$
\frac{\partial O_i}{\partial g_i} = \alpha_i \delta x_i \frac{\partial x_i}{\partial g_i} + t_i \frac{\partial t_i}{\partial g_i} + \frac{\partial x_i}{\partial t_i} - g_i.
$$

It implies that, for any level of public investment in region $j$, returns to immobile factors as well as tax revenue of region $i$ increases with increase in its own public investment. Clearly, $\frac{\partial O_i}{\partial g_i} |_{g_i=0} > 0$, $\forall \alpha_i, \alpha_j \in [0, 1]$ and $\theta \in [0, 1)$. Therefore, it is optimal for each region to spend on public investment, irrespective of the extents of welfare orientation of the regions.

Solving (10a), (10b), (11a) and (11b), we get the equilibrium tax rates, public invest-
ments and allocation of mobile capital, given α_i and α_j (∈ [0, 1]), as follows:

\[
t_i = \frac{(2 - \alpha_i) \delta \left[ (3 - \alpha_j) \delta - (1 - \theta)^2 \right]}{(6 - \alpha_i - \alpha_j) \delta - 2 (1 - \theta)^2} > 0,
\]

\[
g_i = \frac{(1 - \theta) \left[ (3 - \alpha_j) \delta - (1 - \theta)^2 \right]}{(6 - \alpha_i - \alpha_j) \delta - 2 (1 - \theta)^2} > 0, \text{ and}
\]

\[
x_i = 1 - x_j = \frac{[(3 - \alpha_j) \delta - (1 - \theta)^2]}{(6 - \alpha_i - \alpha_j) \delta - 2 (1 - \theta)^2} > 0; \ i, j = 1, 2.
\]

(12)

It is easy to check that \( \frac{\partial g_i}{\partial \theta} < 0 \). That is, higher is the spillover, lower is the public investment in each region. This is due to the well-known free rider problem. When there is spillover of public investment, a region reaps the benefit of public investment provided by its rival, at least partially, without incurring any cost. Higher degree of spillover provides greater incentive to such free riding. Note that, \( g_i = 0 \), if \( \theta = 1 \). That is, if there is perfect spillover, none of the regions spend on public investment.

Substituting the expressions for \( t_i, g_i \) and \( x_i \) from (12) in the expression for \( O_i \), we get

\[
O_i = \frac{[(4 - \alpha_i) \delta - (1 - \theta)^2] [(3 - \alpha_j) \delta - (1 - \theta)^2]}{2 [(6 - \alpha_i - \alpha_j) \delta - 2 (1 - \theta)^2]^2} = O_{i,0}^g, \text{ where superscript \('g' denotes that both the regions spend on public investment. Note that, if only region } i \text{ spends on public investment, stage 3 equilibrium public investments and capital allocation can be obtained by solving (10a), (10b), (11a) and } g_j = 0, \text{ and the corresponding } O_j = \frac{[(3 - \alpha_j) \delta - (1 - \theta)^2]}{2 [(6 - \alpha_i - \alpha_j) \delta - (1 - \theta)^2]^2} = O_{j,0}^g. \text{ Similarly, if only region } j \text{ provides public investment, we get } O_i = \frac{[(4 - \alpha_i) \delta - (1 - \theta)^2]}{2 [(6 - \alpha_i - \alpha_j) \delta - (1 - \theta)^2]^2} = O_{i,0}^g \text{ and } O_j = \frac{[(3 - \alpha_j) \delta - (1 - \theta)^2]}{2 [(6 - \alpha_i - \alpha_j) \delta - (1 - \theta)^2]^2} = O_{j,0}^g. \text{ If none of the regions spend on public investment, } O_i = \frac{[(4 - \alpha_i) (3 - \alpha_j) \delta - (1 - \theta)^2]}{2 [(6 - \alpha_i - \alpha_j) \delta - (1 - \theta)^2]^2} = O_{i,0}^g. \text{ We depict the normal form of the stage 2 game in Figure 4.}

<table>
<thead>
<tr>
<th>Region 1</th>
<th>No public investment</th>
<th>Public investment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( O_{1,0}^g, O_{2,0}^g )</td>
<td>( O_{1,0}^g, O_{2,0}^g )</td>
</tr>
<tr>
<td></td>
<td>( O_{1,0}^g, O_{2,0}^g )</td>
<td>( O_{1,0}^g, O_{2,0}^g )</td>
</tr>
</tbody>
</table>

Figure 4: Decision to spend on public investment
From Figure 4, it is easy to observe that $O_{1_{gg}} > O_{1_{00}}$ and $O_{2_{gg}} > O_{2_{00}}$; $\forall \alpha_1, \alpha_2 \in [0, 1]$ and $0 \leq \theta < 1$. It implies that, if region 2 (region 1) spends on public investment, it is optimal for region 1 (region 2) also to spend on public investment. Moreover, we get $O_{1_{00}} > O_{1_{00}}$ and $O_{2_{00}} > O_{2_{00}}$; $\forall \alpha_1, \alpha_2 \in [0, 1]$ and $0 \leq \theta < 1$. That is, it is optimal for region 1 (region 2) to spend on public investment even if region 2 (region 1) does not provide public investment. Therefore, it is always optimal for a region to spend on public investment, irrespective of (a) whether its rival spends on public investment or not and (b) whether the extents of welfare orientation of the two regions are same or not. In other words, in equilibrium, both the regions spend on public investment irrespective of the regions’ ultimate goals. The intuition behind this result is as follows. If a region spends on public investment, the other region needs to counteract that by undercutting the tax and spending on public investment, since only tax under cutting is sub-optimum from both net tax revenue and social welfare point of view. On the other hand, if a region does not spend on public investment, by providing public investment the other region can increase the tax rate to some extent and still attracts more mobile capital, which in turn leads to higher net tax revenue as well as higher returns to immobile factors.

However, net tax revenue as well as social welfare of each region is lower when the regions spend on public investment compared to that in case none of the regions spend on public investment: $O_{1_{gg}} < O_{1_{00}}, \quad i = 1, 2$. That is, regions are worse-off by spending on public investment. In other words, in equilibrium, both regions spend on public investment and gets lower net tax revenue and lower social welfare compared with that under competition only in terms of tax rates. That is, regions face a Prisoners’ dilemma type of situation while deciding whether to spend on public investment or not and end up with Pareto inferior outcomes.

**Proposition 3:** Each region has unilateral incentive to spend on public investment. In equilibrium, both the regions spend on public investment, irrespective of their ultimate goals - net tax revenue or social welfare, and end up with Pareto inferior outcomes.
It is interesting to note that more welfare oriented region spends more on public investment and charges lower tax rate: $\frac{\partial g_i}{\partial \alpha_i} > 0$ and $\frac{\partial t_i}{\partial \alpha_i} < 0$. Because, perceived benefit from mobile capital is higher to a welfare oriented region than that to a revenue oriented region, since fully revenue oriented region does not care about positive effect of mobile capital on returns to immobile factors. And, allocation of mobile capital in increasing (decreasing) in public investment (tax rate) in that region. On the other hand, due to increase in region $i$’s welfare orientation, region $j$’s public investment falls ($\frac{\partial g_j}{\partial \alpha_i} < 0$) while its tax rate falls by less amount than that of region $i$ ($\frac{\partial t_i}{\partial \alpha_i} < \frac{\partial t_j}{\partial \alpha_i} < 0$). As a result, more mobile capital is allocated to the region that has higher welfare orientation: $\frac{\partial x_i}{\partial \alpha_i} > 0$.

Lemma 3: When regions compete for foreign owned mobile capital both in terms of tax rate and public investment, increase in social welfare orientation of a region induces it (its rival) to spend more (less) in public investment. However, both the regions charge lower tax rate, if there is an increase in welfare orientation of either region. Nonetheless, reduction in tax rate is higher due to increase in its own welfare orientation than that of its rival. Thus, more social welfare oriented region attracts larger share of mobile capital.

Clearly, the implication of welfare orientation on tax rate in case of multidimensional competition is same as that in case of competition only in terms of tax rate. In both the cases, regions can restrict ‘race-to-the-bottom’ in tax rates by moving away from social welfare maximization towards revenue maximization.

Finally, lets turn to the issue of endogenous determination of perceived objective functions of the two regions. Substituting the expressions of $t_i$, $g_i$ and $x_i$ from (12) in the expressions for $NT_i$, $IR_i$ and $SW_i$ we get,

$$NT_i = \frac{2 (2 - \alpha_i) \delta - (1 - \theta)^2} {2 \left[ (6 - \alpha_i - \alpha_j) \delta + 2 (1 - \theta)^2 \right]^{\frac{1}{2}}} \left[ (3 - \alpha_j) \delta - (1 - \theta)^2 \right]^{\frac{1}{2}} \left[ (6 - \alpha_i - \alpha_j) \delta - 2 (1 - \theta)^2 \right]^{\frac{1}{2}}$$

$$IR_i = \frac{\delta \left[ (3 - \alpha_j) \delta - (1 - \theta)^2 \right]^{\frac{1}{2}}} {2 \left[ (6 - \alpha_i - \alpha_j) \delta - 2 (1 - \theta)^2 \right]^{\frac{1}{2}}}$$

$$SW_i = \frac{(5 - 2 \alpha_i) \delta - (1 - \theta)^2} {2 \left[ (6 - \alpha_i - \alpha_j) \delta - 2 (1 - \theta)^2 \right]^{\frac{1}{2}}} \left[ (3 - \alpha_j) \delta - (1 - \theta)^2 \right]^{\frac{1}{2}} \left[ (6 - \alpha_i - \alpha_j) \delta - 2 (1 - \theta)^2 \right]^{\frac{1}{2}}; i, j = 1, 2; i \neq j$$
Note that \[ \frac{∂NT_i}{∂α_i} = -\frac{\delta((3-α_j)δ-(1-θ)^2)^2((2+α_i-α_j)δ-(1-θ)^2)}{(6-α_i-α_j)δ-2(1-θ)^2)^3} < 0 \forall α_i, α_j, θ ∈ [0, 1]. \] Therefore, it is optimal for region \( i \) to set \( α_i = 0 \), if its ultimate target is to maximize net tax revenue, irrespective of the extent of its rival’s extent of welfare orientation \( (α_j) \). That is, it is optimal for a fully revenue oriented region to set the net revenue maximizing tax rate and public investment, no matter what its rival region’s perceived objective function is. In other words, if a region’s ultimate target is to maximize net tax revenue, it is optimal for the region not to delegate the task to decide tax rate and public investment to its manager, irrespective of the other region’s ultimate target and whether the other region delegates or not. It implies that, if both the regions’ ultimate target is to maximize net tax revenue, in equilibrium, none of the regions delegates. Therefore, in case of symmetric regions with respective net tax revenues as ultimate goals, in equilibrium \( \alpha_1^m = \alpha_2^m = 0 \), where superscript \( m \) indicates multidimensional competition, i.e. competition in terms of both tax rate and public investment, between the regions for mobile capital. Lemma 4 summarizes the equilibrium outcomes corresponding to this case.

**Lemma 4:** In case of multidimensional competition for foreign owned mobile capital, if the regions’ ultimate goals are to maximize their respective net tax revenues, the equilibrium incentive parameters, tax rates, public investments, capital allocation, net tax revenues and social welfare are \( \alpha_1^m = \alpha_2^m = 0 \), \( t_1^m = t_2^m = \delta \), \( g_1^m = g_2^m = \frac{1-θ}{2} \), \( x_1^m = x_2^m = \frac{1}{2} \), \( NT_1^m = NT_2^m = \frac{4\delta-(1-θ)^2}{8}, \) and \( SW_1^m = SW_2^m = \frac{5\delta-(1-θ)^2}{8} \), respectively.

Comparing Lemma 2 and Lemma 4, we find that the equilibrium incentive parameters, tax rates and capital allocation in case of multidimensional competition are same as that in case of competition only in terms of tax rates, if the two regions are concerned only about their respective net tax revenues. However, in case of multidimensional competition, net tax revenue and social welfare of both the regions are less than that in case of pure tax competition, as in Proposition 3. To illustrate it further, note that each region chooses the net tax revenue maximizing level of public investment and/or tax rate in equilibrium, since the regions are symmetric. Thus, the two regions provide the same level of public
investment and set the same tax rate. From (10a) and (11a), it is easy to observe that, since 
\( \alpha_1^*m = \alpha_2^*m = 0 \), the effect of public investment in a region on its own tax rate is of same
magnitude as that on its rival’s tax rate, but these effects are of opposite signs.\(^{10}\) Therefore, the equilibrium tax rates remain the same as in case of pure tax competition, since regions
do not differ in terms of level of public investment. As a result, the equilibrium allocation
of mobile capital, returns to immobile factors and gross tax revenue are same in both the
scenarios. However, since regions need to incur some cost to provide public investment,
the equilibrium net tax revenue and social welfare are less in case of multidimensional
competition compared to that in case of pure tax competition.

Now, from (14), we get
\[
\frac{\partial IR_i}{\partial \alpha_i} = \frac{\delta^2 [(3-\alpha_j)\delta-(1-\theta)^2]^2}{[6-\alpha_1-\alpha_j] \delta - \delta^2 (1-\theta)^2} > 0 \forall \alpha_i, \alpha_j, \theta \in [0,1], \text{ since } \delta > 1. 
\]
That is, returns to immobile factors in region \(i\) is increasing in extent of social welfare
orientation of region \(i\), irrespective of the extent of region \(j\)’s social welfare orientation.
However, net tax revenue of a region is decreasing in its extent of welfare orientation:
\[
\frac{\partial NT_i}{\partial \alpha_i} < 0 \forall \alpha_i, \alpha_j, \theta \in [0,1]. 
\]
Therefore, whether social welfare of a region is increasing in its extent of welfare orientation or not that depends on relative magnitudes of the effects
of its extent of welfare orientation on its net tax revenue and returns to immobile factors,
since \(SW_i = IR_i + NT_i\). Differentiating both sides of (15) with respect to \(\alpha_i\) we get,
\[
\frac{\partial SW_i}{\partial \alpha_i} = -\frac{\delta [(3-\alpha_j)\delta-(1-\theta)^2]^2 [1+\alpha_1-\alpha_j] \delta - (1-\theta)^2}{[6-\alpha_1-\alpha_j] \delta - \delta^2 (1-\theta)^2} \]
Clearly, if \(\alpha_j = 0\), \(\frac{\partial SW_i}{\partial \alpha_i} < 0 \Rightarrow \alpha_i^m = 0\), \(i,j = 1,2\). That is, if any one of the two regions set the net tax revenue maximizing
tax rate and public investment, it is optimal for the other region also to do so even if its
ultimate goal is to maximize social welfare. Therefore, \((\alpha_1^m = \alpha_2^m = 0)\) is an equilibrium,
even if the ultimate goal of the regions are to maximize respective social welfare.

From the expression for \(\frac{\partial SW_i}{\partial \alpha_i}\), it is easy to check that (i) \(\frac{\partial SW_i}{\partial \alpha_1} > 0\), if (a) \((1 + \alpha_1 - \alpha_2) \delta < (1 - \theta)^2\); and (ii) \(\frac{\partial SW_2}{\partial \alpha_2} > 0\), if (b) \((1 + \alpha_2 - \alpha_1) \delta < (1 - \theta)^2\). However, note that (a) and
(b) together implies that \(\delta < (1 - \theta)^2\), which is impossible since \(\delta > 1\) and \(0 \leq \theta < 1\).
Therefore, in equilibrium, both \(\alpha_1\) and \(\alpha_2\) can not be positive. And, we have already shown
that if \(\alpha_1 = 0\), \(\alpha_2^*m = 0\), vice-versa. Clearly, \((\alpha_1^m = \alpha_2^m = 0)\) is the only subgame perfect

\(^{10}\)\(\frac{\partial t_i}{\partial g_i} \big|_{(\alpha_1=0,\alpha_2=0)} = -\frac{\partial t_j}{\partial g_i} \big|_{(\alpha_1=0,\alpha_2=0)}.\)
equilibrium, though the regions’ ultimate goals are to maximize respective social welfare. The above analysis also implies that \((\alpha_1^{*m} = \alpha_2^{*m} = 0)\) is the only equilibrium even if one region wants to maximize net tax revenue while the other region wants to maximize social welfare. Because, if region \(i\) aims to maximize its net tax revenue, it never sets \(\alpha_i > 0\). And, if \(\alpha_i = 0\), it is optimal for region \(j\) to set \(\alpha_j = 0\) even if region \(j\)’s target is to maximize its social welfare. Therefore, we can say that, irrespective of the regions’ ultimate goals, it is always optimal for each region to perceive its net tax revenue as the objective function while deciding its strategies to compete for mobile capital. In other words, only if the ultimate target of a region is to maximize its social welfare, in equilibrium, it delegates the task to decide the tax rate and level of public investment to its manager by offering a fully revenue oriented contract.

**Proposition 4:** (a) In case of multidimensional competition for foreign owned mobile capital, net tax revenue maximizing tax rate and public investment maximizes social welfare. (b) In equilibrium, each region perceives its net tax revenue as the objective function while deciding the tax rate and level of public investment, irrespective of whether the region wants to maximize its social welfare or net tax revenue and whether the regions have different goals or not.

Since \(\alpha_1^{*m} = \alpha_2^{*m} = 0\) holds true irrespective of the regions’ ultimate goals, the equilibrium tax rate, public investment, allocation of mobile capital, net tax revenue and social welfare remain same as in Lemma 4, even if the regions are concerned about their respective social welfare or if the ultimate goals of the two regions are different from each other. From the above discussion it appears that, whatever be the ultimate goals of the regions, it is always optimal for each region to perceive its net tax revenue as the objective function in both multidimensional competition as well as pure tax competition. Therefore, consideration of fully revenue oriented regions seems to be more appropriate while analyzing the implications of competition for mobile capital among regions.


5 Conclusion

This paper endogenizes the governments’ objective functions and the decision to spend on productivity enhancing public investment, by developing a model of interregional competition for foreign owned mobile capital. Considering two competing regions, it shows that it is always optimal for the competing regions to choose their respective net tax revenue maximizing strategies, while competing for mobile capital, even if its ultimate goal is to maximize social welfare, irrespective of whether the competition is only in terms of tax rates or both public investment as well as tax rate. This result remains valid even if the two regions are asymmetric in terms of their ultimate goals. It also demonstrates that the competing regions can restrict race-to-the-bottom in tax rates by deviating away from social welfare to net tax revenue. Further, it demonstrates that the regions have unilateral incentive to spend on public investment, unless the spill over is perfect. In equilibrium, both the regions spend on public investment and end up with Pareto inferior outcomes. These are new results.
References


